Matched field inversion of geoacoustic properties from towed array data in shallow water.¹

O. Lotsberg² and S.M. Jesus

UCEH - University of Algarve, PT-8000 Faro, Portugal

SUMMARY

This work reports the inversion results obtained on part of the data collected during a sea trial that took place in the Pantelleria Bank, south of Sicily, in March 1994. During that experiment the source was emitting acoustic tones at frequencies between 100 and 200 Hz and the towed array had 40 hydrophones at 4 m spacing. The array shape was measured continuously during the trial, and was considered known in the forward model. After adjusting the forward model to correct water depth and layer thickness inversion for p-velocity gave consistent results in some part of the survey when compared to the available ground truth measurements made during the cruise. The results obtained at frequencies around 150 Hz have a higher accuracy and are more consistent over time than those obtained at lower frequencies. The results show also that a mismatched forward model will lead to, sometimes consistent, biased inversion results. As expected from simulated studies, shear velocities have very little influence on the acoustic field, and had no influence on the inversion.

1. INTRODUCTION

This EC financed project has the goal of investigating the feasibility of a quantitative estimate of seafloor geophysical/geoacoustic parameters (compressional and shear velocities, attenuation, density, ...) by using acoustic remote sensing techniques. In particular, this project was concerned with the use of ship-towed instrumentation, that would allow seafloor properties to be estimated from a moving ship in a survey fashion.

The main idea of the project was that of inverting the acoustic field produced by a known source and measured at a receiving array in order to identify a physical seafloor model. Such general techniques are referred to in the literature as 'full field' inversion methods. The full-field inversion schemes can also be referred to as Matched Field Inversion.

Matched Field Inversion has shown successful in several source receiver configurations. In particular, long aperture synthetic arrays [1] or vertical arrays spanning the whole water column [2]. The experimental configuration considered in this project however, is more adapted to survey applications, and is much more challenging from the point of view of

¹ Funded by the European Community under contract MAS2-CT920022
² Present address: NTNU - Norwegian University of Science and Technology, 7034 Trondheim, Norway
bottom parameter identification, because of the short array, the big distance between the bottom and the array/source and because of the system movement the towing will cause.

2. BACKGROUND

The deterministic sound pressure at receiver location \((r, z)\) is modeled as the solution of the wave equation for a narrowband point source exciting a horizontally stratified range-independent fluid-elastic environment. It is given by the Green’s function inverse zero-order Hankel transform:

\[
p_l(\omega_k, r, z_1, z_0; \gamma) = \int_0^\infty g(\kappa, \omega_k, r, z_1, z_0; \gamma, z_0) J_0(\kappa r) d\kappa
\]

(1)

where \(l\) denotes the \(l^{th}\) array sensor, \(\omega_k\) is the \(k^{th}\) frequency bin, \(z_0\) is the source depth and \(\gamma\) is a vector containing all the pertinent environmental parameters. OASES [3] (transmission loss module) is designed for the purpose of solving this equation numerically. When the receivers are not at the same depth (tilted array) the acoustic pressure \(p(\bullet; \gamma)\) is calculated by OASES in the following way: Specify all the receiver ranges and all the receiver depths; this gives a matrix (\(N\) times \(N\), where \(N\) is number of receivers) as output. The tilted array field is the diagonal of this matrix. The computational effort of solving (1) consists of two parts: First, determine the depth dependent Green’s function \((g(\bullet))\), second, do the inverse Hankel transform. The second part must be done for each specified depth \((z)\); which means one time for a horizontal array, and \(N\) times for a tilted array.

At time snapshot \(n\), the acoustic pressure field received by an array of \(L\) sensors can be modeled as a multivariate complex normally distributed random variable:

\[
y_n(\omega_k, \gamma_T) = b_n(\omega_k) p(\omega_k, \gamma_T) + \epsilon_n(\omega_k), k = 1, \ldots, K
\]

(2)

where \(\epsilon\) is the sensor noise assumed to be zero mean and uncorrelated both in time and from sensor to sensor. The scalar number \(b_n\) is a complex random variable that accounts for the non deterministic amplitude variation at the receiver due to environmental inhomogeneities and fluctuations that are not included in the sensor noise. Subscript \(T\) denotes the true value of the parameter under estimation.

To decide which forward model that fits the real data best, we have in this work used the so called Conventional Matched Filter (CMF) [4].

\[
\Phi(\gamma) = \left| 1 - \sum_{k=1}^{N_{rec}} \tilde{p}_k \hat{q}_k \right|
\]

(3)

where \(\tilde{p}, \hat{q}\) are the real and calculated (from OASES) data respectively normalized to unity and \(*\) is the complex conjugate. This gives us the following estimate of the environmental parameters:
\[ \hat{\gamma}_r = \arg \min_{\gamma} \phi(\gamma) \] (4)

where \( \gamma \) is allowed to vary within pre-defined intervals.

3. INVERSION STRATEGY

The ambiguity surface of the function to optimize is multidimensional, and is suspected to have several local minima and maxima. This excludes the use of classical gradient-based search methods. The strategy used for determining the inverse solution in this work was the so called Genetic Algorithm. This is a global search technique which avoids the exhaustive search over the whole parameter space, while maintaining the ability of escaping from a local minima. In the following familiarity with the GA machinery and GA jargon is assumed. The implementation of the algorithm was done by Gerstoft, through the program called SAGA [5]. For tuning the GA parameters the indications in Gerstoft [6] was used:
The population size \( q \) was chosen to be 20 - 40 (dependent on which part of the survey); large enough to represent several minima, and the reproduction size \( f \) was 0.5, so that the most fitted half of the individuals always stay in the population. Crossover rate \( p_c \) was 0.8, the mutation rate \( p_m \) was 0.05; higher than usual in GA, and the number of forward computations was 1500-2500 (dependent on which part of the survey).
The inversion was done on an Alpha AXP 190 MHz server, with four parallel processors. On this computer it was possible to invert approximately 24 seconds of data per hour. In this work it was put no effort in trying to decrease the inversion time.

4. EXPERIMENTAL SETUP

The experimental setup was as follows: The survey ship towed at the same time, on two independent cables, an acoustic source and an array of 40 receivers with 4 meters spacing. The nominal depth of the source and the receivers were measured continuously during the trial by non-acoustical means. The measurements showed a relatively constant source depth of 40 meters, while the receiver depth varied both with time and receiver position; the array was clearly oscillating up and down in time and the shape of the array was not a horizontal line but a bowed curve with the receiver closest to the ship as the uppermost receiver. The measurements showed larger movements than what seemed realistic (1 m/s in the z-direction), and we therefore decided to use an average (1 minute of measurements) of the measured array shape in the inversion. The distance between the source and the first receiver was measured acoustically approximately every 2 hours and showed a relatively constant value of 535 meters. The transmitted frequency varied from one survey to another, and was always between 100 and 200 Hz. The water depth was also measured continuously, and the sound velocity profile of the water column was measured once a day by CTD. The CTD measurement showed an almost constant sound velocity profile of 1507 - 1510 m/s.

5. RESULTS

To evaluate the performance of the method it is necessary to have available an independent measurement of the same geophysical properties that are estimated with the acoustic
system. The approximate result of this 'ground truth' measurement is summarized in Table 1.

<table>
<thead>
<tr>
<th></th>
<th>p-velocity</th>
<th>s-velocity</th>
<th>density</th>
<th>thickness</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>m/s</td>
<td>m/s</td>
<td>g/cm³</td>
<td>m</td>
</tr>
<tr>
<td>layer 1</td>
<td>1550</td>
<td>230</td>
<td>1.49</td>
<td>10</td>
</tr>
<tr>
<td>layer 2</td>
<td>1610</td>
<td>290</td>
<td>1.88</td>
<td>5</td>
</tr>
<tr>
<td>layer 3</td>
<td>1700</td>
<td>360</td>
<td>2.4</td>
<td>5</td>
</tr>
</tbody>
</table>

Table 1 'ground truth' measurement

Only the last 36 hydrophones were used in the inversion because of a disfunction of hydrophone number 4, which leaves a total array length of 140 meters. The inversion was done in two steps. First, a short period of data was inverted. Many parameters (water depth, p-velocity and layer thickness) were included in this first inversion. In the next step the values of the water depth and layer thickness from step one was used in the forward model. In this step only p-velocity (search from 1500 m/s to 2000 m/s divided into 64) was included in the inversion, and long periods of data was inverted. Shear velocity was not included in the inversion.

In Table 2 and Figure 1 the results of three consecutive runs from the two uppermost layers (15 meters and 5 meters respectively) are reported. Notice that the velocity given in the table is the median and not the mean. This is done because, as one can see from Figure 1, when the algorithm fails the values are often far from realistic values. If one takes the median (the number in the middle) instead of the mean, these unrealistic jumps have less significance.

<table>
<thead>
<tr>
<th>Time</th>
<th>Freq.</th>
<th>Vel. layer1</th>
<th>std. layer1</th>
<th>Vel. layer2</th>
<th>std. layer2</th>
</tr>
</thead>
<tbody>
<tr>
<td>09:48:55</td>
<td>150 Hz</td>
<td>1591 m/s</td>
<td>66.0 m/s</td>
<td>1730 m/s</td>
<td>113.2 m/s</td>
</tr>
<tr>
<td>09:54:05</td>
<td>175 Hz</td>
<td>1532 m/s</td>
<td>33.4 m/s</td>
<td>1794 m/s</td>
<td>65.4 m/s</td>
</tr>
<tr>
<td>09:59:14</td>
<td>200 Hz</td>
<td>1557 m/s</td>
<td>147.6 m/s</td>
<td>1675 m/s</td>
<td>56.2 m/s</td>
</tr>
</tbody>
</table>

Table 2 Median of p-velocity of two uppermost layers (15 and 5 meters respectively)

These results look quite good, and if all the results had looked like this, the method would have been very promising. But unfortunately there are several reasons why the conclusion is not so positive. The first is that even if these results look good with values not far from the 'ground truth' and with low standard deviation, the values are not continuous between the three different parts. The frequency is changed between the three parts (150, 175 and 200 Hz), and the forward model must therefore also be changed. This leads to, as one can see at around 580 meters, a change in the inversion result, which is of course not realistic.

In Figure 2 another inversion result (p-velocities of three uppermost layers) is reported. In this run the water depth is not constant according to the 'ground truth', but in the forward model it is nevertheless kept constant. This leads to an unrealistic gradual change in the inverted p-velocity in the second layer. One can also see that the inversion values of the third layer suddenly changes from a realistic value with low standard deviation to an unrealistic value. The example shows that one has to be very careful if one wants to keep anything in the forward constant.
At frequencies lower than 125 Hz the results were usually very bad, with high standard deviation. None of these runs are reported here.

Figure 1 Inversion result (p-velocity) of three consecutive runs of the two uppermost layers (15 and 5 meters respectively). The frequency was 150 Hz, 175 Hz and 200 Hz for layer1, f109, 150 Hz, wd: 134 m
layer2, f109, 150 Hz, wd: 134 m
layer3, f109, 150 Hz, wd: 134 m

Figure 2 Inversion result (p-velocity of three uppermost layers). According to the 'ground truth' the water depth is not constant, but in the forward model it is nevertheless set to 134 meters (mean value). The mismatch between forward model and 'ground truth' leads to an unrealistic change in p-velocity in the second layer and unstable inversion of layer 3.
5. CONCLUSIONS

If one can find a forward model where the important parameters which are not included in the inversion are reasonable, the inversion results for the p-velocity look quite good. But the inversion results show also that a stable mismatch in the forward model can lead to biased and stable values in the inversion. When one parameter is gradually changing in the real environment, and is kept constant in the forward model, one of the parameters included in the inversion will gradually change to compensate for this. Model mismatch can also be the reason why the result in some part of the run can look stable and good and then suddenly become unstable and bad. At frequencies lower than 125 Hz the results had very high standard deviation.

6. FURTHER WORK

A priori knowledge (e.g. increasing velocity for increasing depth) should be included. This would reduce the search space and give more physical reliable results. Some kind of smoothing of the data should be included. It is difficult to know how the smoothing should be done in an optimal way; i.e. how to get rid of the noise without destroying too much of the information in the signal. Some preliminary investigation with the ML objective function have shown quite promising results, and in future work this function should be used.

ACKNOWLEDGMENTS

This work was funded by the European Community under contract MAS2-CT920022. We will also like to thank Peter Gerstoft for helpful advice regarding the Genetic Algorithm and use of the program SAGA.

REFERENCES


