AIP/123-QED

Three-dimensional model benchmarking for cross-slope wedge propagation

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(Dated: May 11, 2017)

INTRODUCTION

Modeling of acoustic fields in three-dimensional waveguides had been an active field of research for many years;[2,6,7,11,23] the wedge problem has been an important reference in this context given the availability of an analytical solution.[3,4,20,22] Despite the apparent simplicity of the wedge waveguide the corresponding solution has revealed many interesting features of three-dimensional propagation, such as horizontal refraction and mode coupling. Experimental evidence also indicates important features of out-of-plane propagation, i.e. rays that propagate up-slope before connecting a source to a receiver.[15,17] The issue of cross-slope wedge propagation has been discussed in detail in[9,18,19] for experimental data and for analytical predictions of adiabatic and non-adiabatic propagation using the parabolic equation model 3DWAPE;[17] the model was able to provide extremely accurate predictions in all cases. The main goal of the work presented here is to develop a benchmarking of cross-slope wedge propagation with three-dimensional models, relying on normal mode and ray tracing theory; the analytical cases and the experimental data for which 3DWAPE was already benchmarked are again considered. Such benchmarking is expected to identify the advantages, accuracy and limitations of models that rely on approximations, different to the one used by the parabolic method; the discussion is also important for the development of applications in more general three-dimensional waveguides. This work is organized as follows: the geometry of the wedge problem is presented in section 2, together with a compact description of the analytical solutions and of the tank scale experiment; section 3 describes the three-dimensional models used in the benchmarking, while section 5 presents the results of benchmarking. The main conclusions and future work are presented in section 6.

CROSS-SLOPE WEDGE PROPAGATION

The geometry of cross-slope wedge propagation is shown in Fig. 1; bottom slope is given by α, the source is located at the position $(0, 0, z_s)$ and the array is aligned horizontally along the Y axis; sound speed in the water is constant, and the bottom is homogeneous, characterized by a given density, compressional speed and attenuation; bottom shear is not considered. The following sections briefly discuss the analytical solutions (which are available at the Ocean Acoustics Library website[8]), together with the scale experiment. Specific values of wedge parameters are presented in section 5 and are further used for benchmarking.

ADIABATIC PROPAGATION

Three-dimensional propagation in any waveguide can be expanded on a basis of local modes, with modal amplitudes obeying a set of differential equations containing crossed (non-diagonal) terms. For the specific conditions of the wedge the local modes can be calculated using the formalism of the Pekeris waveguide; the adiabatic approximation further relies on neglecting non-diagonal terms, a condition that is valid for "small" values of α, altough this assumption is difficult to define for an arbitrary set of wedge parameters. When mode coupling is neglected the original system of equations for the modal amplitudes is simplified and can be solved analytically.[9]

NON-ADIABATIC PROPAGATION

The analytical solution for the wedge problem is based on the method of images; generally speaking, the contribution of each image can be represented in terms of a Bessel function expansion inside an improper integral; numerical implementation of the solution is generally intensive because the convergence of the series is slow, and worsens when small α are considered; in this case the image solution can be replaced with the much faster adiabatic solution.

EXPERIMENTAL DATA

Experimental data for benchmarking was obtained from a scale experiment, that was developed in July 2007 at the LMA-CNRS laboratory in Marseille, using an indoor shallow water tank with dimensions 10-m long, 3-m wide and 1-m deep.,[18] A bottom sloped half-space was made with river sand, whose properties were measured carefully. Five-cycle pulses with Gaussian envelopes were transmitted in the watercolumn overlying the sandy bottom for three different depths of the acoustic source. Time series were recorded in a cross-slope direction at consecutive ranges from the source with step Δr = 0.1 m up to 5 m. In order to compare the results of the scale experiment with model predictions the following conventions will be used in section 5:

- Experimental frequencies in kHz will be indicated as model frequencies in Hz (for instance, an experimental frequency of 150 kHz will be indicated as a model frequency of 150 Hz).
- Experimental depths in cm will be indicated as model depths in m (for instance, an experimental source depth of 8.3 cm will be indicated as a model source depth of 8.3 m).
- Experimental ranges in m will be indicated as model ranges in km (for instance, an experimental range of 5 m will be indicated as a model range of 5 km).
- Attenuation units in dB/λ and sound speeds do not require any conversion.

THE MODELS

The models used for benchmarking were KRAKEN (a normal mode model), Bellhop3D and TRACEO3D (both based on ray tracing). All models are described in the following sections.

KRAKEN

KRAKEN is a well known model based on normal mode theory,[12] which is distributed as part of the Acoustic Toolbox.[8] KRAKEN is often used for two-dimensional calculations, but three-dimensional calculations in a variable bathymetry can be developed through the combination of local modes along different bearings starting from the source; such calculations rely on an adiabatic approximation, which can account for horizontal refraction.[12]

BELLHOP3D AND TRACEO3D

Bellhop3D is a recent three-dimensional extension of the Bellhop ray model;[10, 13] both Bellhop and Bellhop3D are also part of the Acoustic Toolbox. Correspondingly, TRACEO3D is a recent three-dimensional extension of the TRACEO ray model.[14] Generally speaking, Bellhop3D and TRACEO3D produce a prediction of the acoustic field in two steps: first, the Eikonal equation is solved in order to provide ray trajectories; second, ray trajectories are considered as the central axes of Gaussian beams, and the acoustic field is calculated as the coherent superposition of beam influences. Bellhop3D and TRACEO3D rely on different numerical strategies to proceed along the two steps; yet, a detailed description of the corresponding details is far beyond the main purpose of this work and will be addressed independently in the future.

BENCHMARKING

The wedge waveguides discussed in[9, 19] and[18] are here again considered for benchmarking with the most recent versions of KRAKEN, Bellhop3D and TRACEO3D, looking for a common reference with 3DWAPE; it should be noticed that, in a certain sense, there are an "old" and "recent"versions of 3DWAPE, since in between the references cited above the model was progressively updated; the discussion presented here is more focused on testing the accuracy of predictions, rather than discussing which version of 3DWAPE was used. Waveguide parameters and corresponding values are summarized in Table 1 and Table 2; the parameters for the non-adiabatic case correspond to the well known 3-D ASA wedge benchmark. The benchmarking is particularly demanding for all models because field coherence is expected to be properly predicted for large values of $R/D(0)$; for the ray tracing models is additionally demanding because the frequencies in all cases are below the threshold of validity of ray theory, given by:[5]

$$f > 10\frac{c_w}{D(0)} \ . \tag{1}$$

Ray model accuracy at low frequencies can be improved by including the beam displacement approximation, in which a ray propagates along a boundary before being reflected back to the watercolumn.[1] Beam displacement has been discussed for two-dimensional modeling in the case of flat and sloped bottoms;[21,24] for a Gaussian beam model the approximation needs to be written in terms of beam parameters, and it is implemented in Bellhop only for a flat boundary (unfortunately, it seems to be undocumented). It will be shown later that beam displacement can be replaced with a much simpler approach, namely by using an "equivalent" value of c_b, lower than the real one. The physical explanation fot this remains unclear, as well as the criterion that can lead to a proper choice of the equivalent c_b; both issues are expected to be addressed in future discussions. Predictions with the KRAKEN model were produced in the analytical cases using a set of $N \times N$ nodes, which defined a mesh based on Delaunay triangulation; an optimal value of N was selected once convergence was achieved; horizontal refraction was taken into account to improve accuracy. KRAKEN three-dimensional calculations rely on an adiabatic approximation; therefore, it was not surprising to find that the corresponding predictions for the experimental data did not produce accurate results; for this reason only Bellhop3D and TRACEO3D results are shown in this case. To generate a prediction with the ray models it was produced first a prediction with KRAKEN for a flat case; an estimate of ray aperture (i.e. of $[-\theta_{\max}, \theta_{\max}]$, where θ stands for ray elevation - the angle relative to the XY plane) was produced by minimizing the standard deviation between the transmission losses calculated by KRAKEN and by the ray model along a single azimuth (i.e. the angle ϕ on the XY plane relative to the X axis). With $\theta_{\min}$ fixed a prediction was produced by the ray model along several azimuths, starting from $\phi = 90°$, until convergence was achieved. Rays launched along a common azimuth quickly spread over the wedge in different directions depending on the original ray elevation; therefore, a value of $N_\theta = 401$ was used to ensure a proper superposition of beam influences. Values of $\theta_{\max}$ and $[\phi_{min}, \phi_{\max}]$ are indicated in all cases.

ANALYTICAL SOLUTIONS

Benchmarking results with KRAKEN, Bellhop3D and TRACEO3D for adiabatic propagation are shown in Fig. 2; to ease the comparisons the entire range is divided in two intervals, one from 0 to 4 km and one from 4 to 8 km. The KRAKEN prediction was obtained with N = 35; parameters for the ray models were given by $\theta_{\max} = 38°$ and $\phi \in [90°, 100°]$. Clearly KRAKEN produces the best match in the first interval, while in the second interval the prediction starts to exhibit a progressive phase mismatch; Bellhop3D and TRACEO3D produce very similar predictions along the two intervals, but they tend to follow KRAKEN's phase, rather than the reference one; Bellhop3D and TRACEO3D also produce less smooth predictions, a fact which is believed to be due to the low value of frequency.

Benchmarking results with KRAKEN, Bellhop3D and TRACEO3D for non-adiabatic propagation are shown in Fig. 3; again, to ease the comparisons the entire range is divided in two intervals, one from 0 to 9 km and one from 9 to 18 km; beyond 18 km propagation is dominated by the first mode. The KRAKEN prediction was obtained with N = 41; parameters for the ray models are given by $\theta_{\max} = 29°$ and $\phi \in [90°, 110°]$. This time KRAKEN's prediction is accurate only up to 5 km, after that phase and

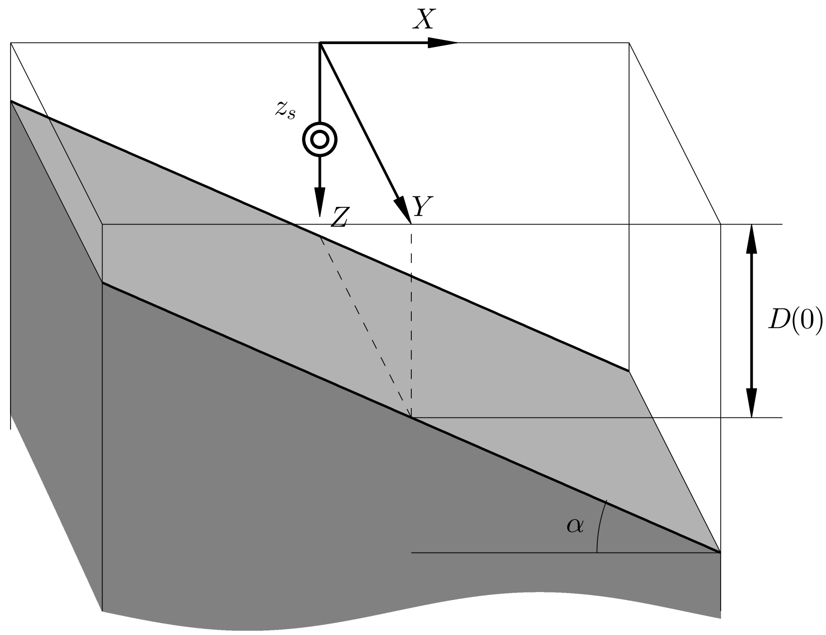

Figure 1: Cross-slope wedge geometry; α stands for the bottom slope, $D(0)$ is bottom depth at the source position, the double circle indicates the position of the acoustic source, the hydrophone array is located along the Y axis.

Table 1: Wedge parameters and corresponding symbols.

Parameter	**Symbol**
Bottom slope	α
Source frequency	f
Depth at source position	$D(0)$
Source depth	z_s
Receiver depth	z_r
Maximal range	R
Water sound speed	c_w
Bottom compressional speed	c_b
Bottom compressional density	ρ_b
Bottom compressional attenuation	α_b

Table 2: Wedge parameters for the different benchmarking cases and corresponding units.

	Adiabatic	Non-adiabatic	LMA-CNRS Experiment			Units
α	0.5	2.86	4.55	4.55	4.55	degrees
f	50	18	150	150	150	Hz
$D(0)$	90	200	44.4	44.2	44.1	m
R	8	25	5	5	5	km
z_s	10	100	8.3	17.5	25.4	m
z_r	10	30	9.3	8.5	8.5	m
c_w	1500	1500	1488.2	1488.7	1488.7	m/s
c_b	2000	1700	1700	1700	1700	m/s
ρ_b	2	1.5	1.99	1.99	1.99	kg/m^3
α_b	0.5	0.5	0.5	0.5	0.5	dB/λ

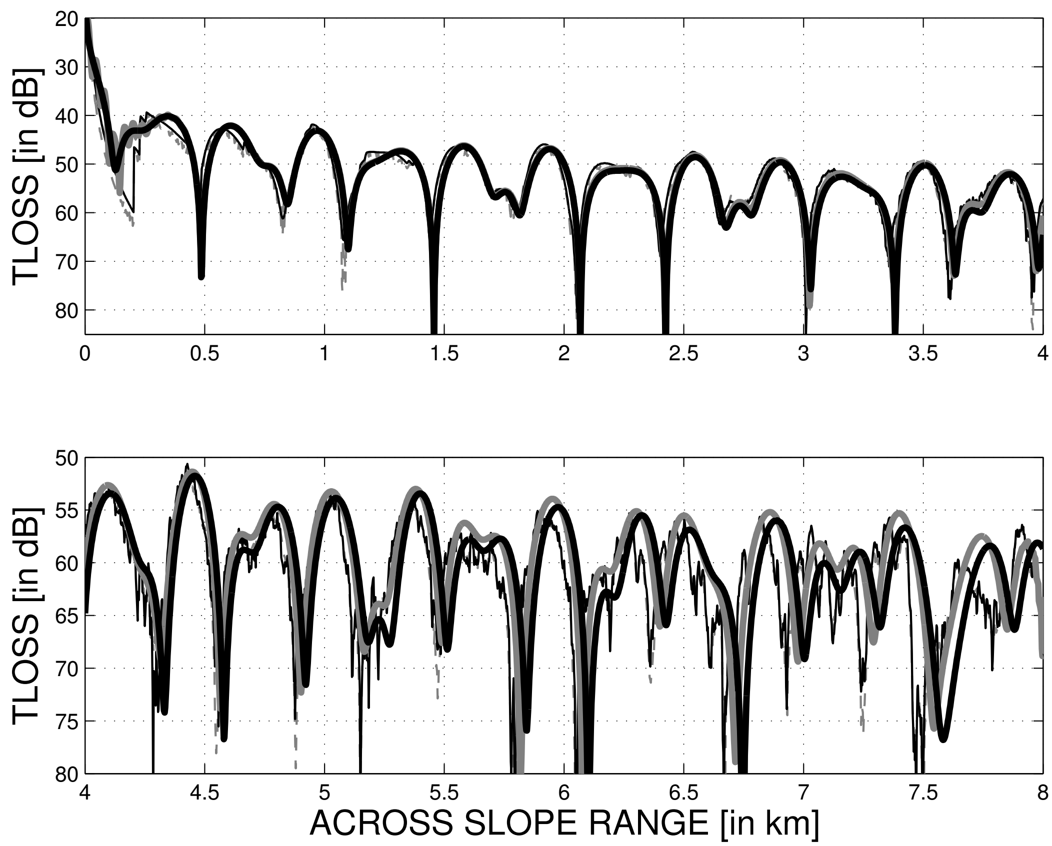

Figure 2: Benchmarking results for the analytical adiabatic case: interval from 0 to 4 km (top), and from 4 to 8 km (bottom); color convention: black, reference; gray, KRAKEN; thin black, Bellhop3D; dashed gray, TRACEO3D.

amplitude mismatches steadily grow and the model prediction deviates from the reference solution; this is believed to be due to the failure of the adiabatic approximation used by KRAKEN, which is no longer valid for the given wedge slope. Despite the low value of frequency Bellhop3D produces a surprisingly accurate match, both im amplitude and phase, while TRACEO3D produces a good amplitude prediction, but steadily accumulates a phase mismatch along range.

EXPERIMENTAL DATA

The scale tank experiment considered three different positions of the acoustic source (see Table. 2), called "H1","H2" and "H3", for the upper, middle and lower positions, respectively. Experimental data for H1 was acquired not only at 150 kHz, but also at 122, 141.6, 161.13 and 180.05 kHz[1]. An important aspect of the experimental parameters indicated in Table. 2 is that they were optimized for 3DWAPE to produce the best predictions of transmission loss. The analytical solution for non-adiabatic propagation can be used here for an important test, namely, to compare the analytical predictions with the experimental data using the parameters shown in the table. Surprisingly, such predictions (not shown here) exhibit a slight phase mismatch in all cases. It was found that the phase mismatch could be mitigated (and the accuracy of prediction greatly improved) by using a value of c_b, lower than the one indicated in Table. 2. The reason for this remains unclear; it is believed to be an important feature of model ambiguity, in the sense that different three-dimensional models will not necessarily provide a common set of waveguide parameters when optimized to the same set of data. To provide a proper reference for the benchmarking of Bellhop3D and TRACEO3D with the experimental data the following strategy was adopted: analytical solutions were compared with the experimental data, for all frequencies and source positions, lowering the value of c_b until the best match was achieved; the value of c_b that provided the better match along frequencies and source position was used as the "equivalent" c_b to used for benchmarking. In this way the parameters for the ray models were given by $\theta_{\max} = 24°$, $\phi \in [90°, 110°]$ and c_b = 1656 m/s.

Benchmarking results with Bellhop3D and TRACEO3D for the upper position of the acoustic source and different frequencies are shown in Fig. 4. They indicate that as long as the eqivalent value of c_b is used Bellhop3D and TRACEO3D are able to produce accurate predictions of the experimental data along the different frequencies.

Benchmarking results with Bellhop3D and TRACEO3D for the three positions of the acoustic source and 150 Hz are shown in Fig. 5. One more time Bellhop3D and TRACEO3D produce similar predictions, but as source depth increases both predictions start to deviate from the reference, implying that the strategy of using a lower c_b is not always effective.

CONCLUSIONS AND FUTURE WORK

A three-dimensional benchmarking for cross-slope wedge propagation, with a normal mode model and two ray tracing models, was discussed in detail for analytical solutions and for experimental data. Generally speaking, the results are encouraging. The normal mode model KRAKEN was able to provide smooth and accurate predictions for the adiabatic case; despite the low frequencies considered in all cases the recent ray models Bellhop3D and TRACEO3D were found to provide accurate predictions, when the reference compressional sound speed was substituted with an equivalent value. Such substitution is believed to be a simpler way to account for beam displacement, although this hypothesis requires further research. As shown by the comparison between the source images solution and experimental data (not shown in this discussion) this issue is also related to model ambiguity, in the sense that optimization to the same data with different three-dimensional models can provide different waveguide parameters. Bellhop3D was found to

[1]Experimental frequencies; for modeling purposes a factor 1000:1 is applied, thus 122 kHz becomes 122 Hz, 141.6 kHz becomes 141.6 Hz, and so on.

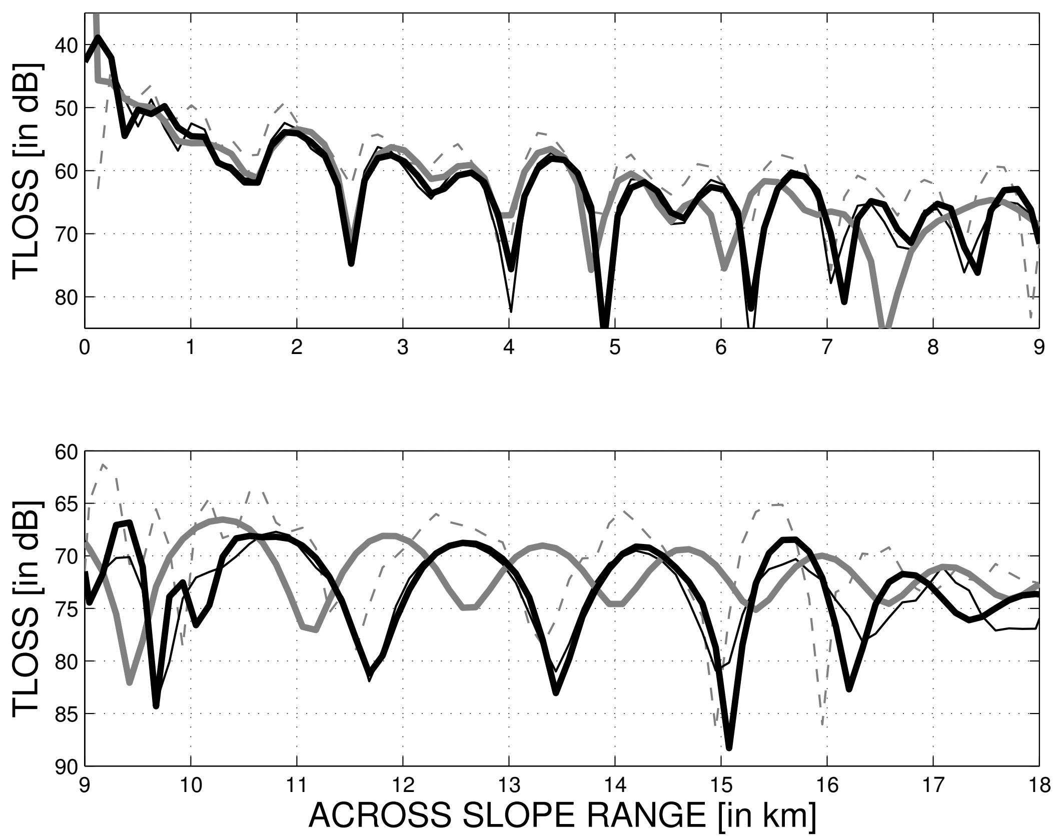

Figure 3: Benchmarking results for the analytical non-adiabatic case: interval from 0 to 9 km (top), and from 9 to 18 km (bottom); color convention: black, reference; gray, KRAKEN; thin black, Bellhop3D; dashed gray, TRACEO3D.

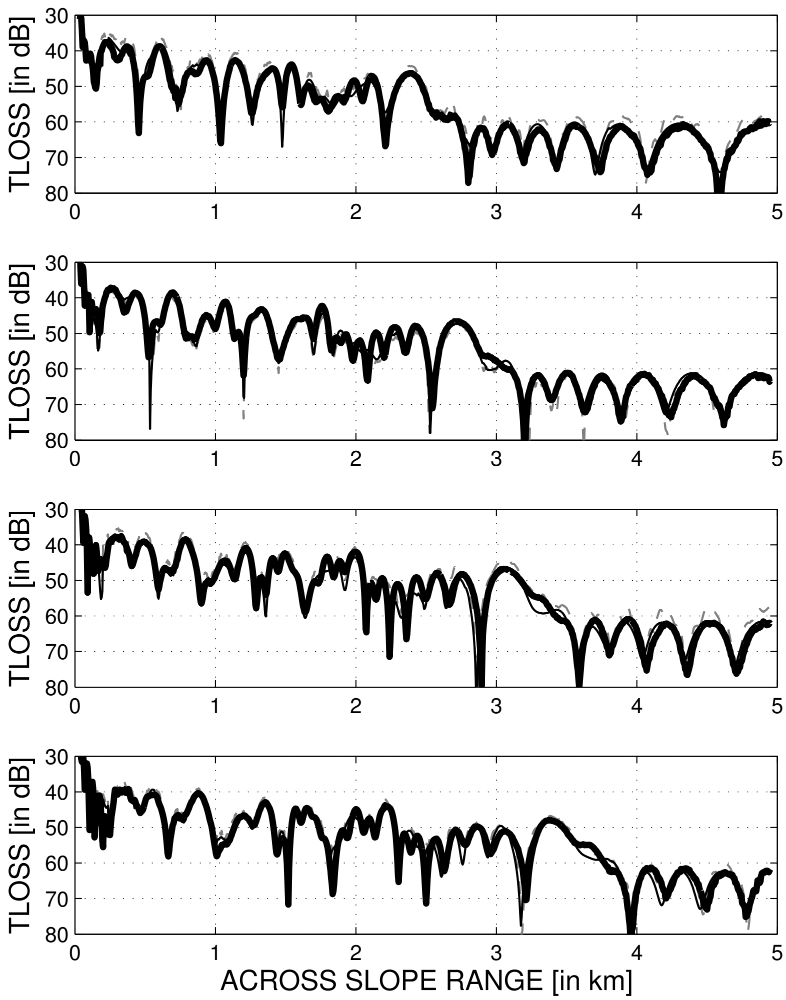

Figure 4: Benchmarking results for experimental data and the upper position of the acoustic source; from top to bottom: 122, 141.6, 161.13 and 180.05 Hz; color convention: black, reference; thin black, Bellhop3D; dashed gray, TRACEO3D.

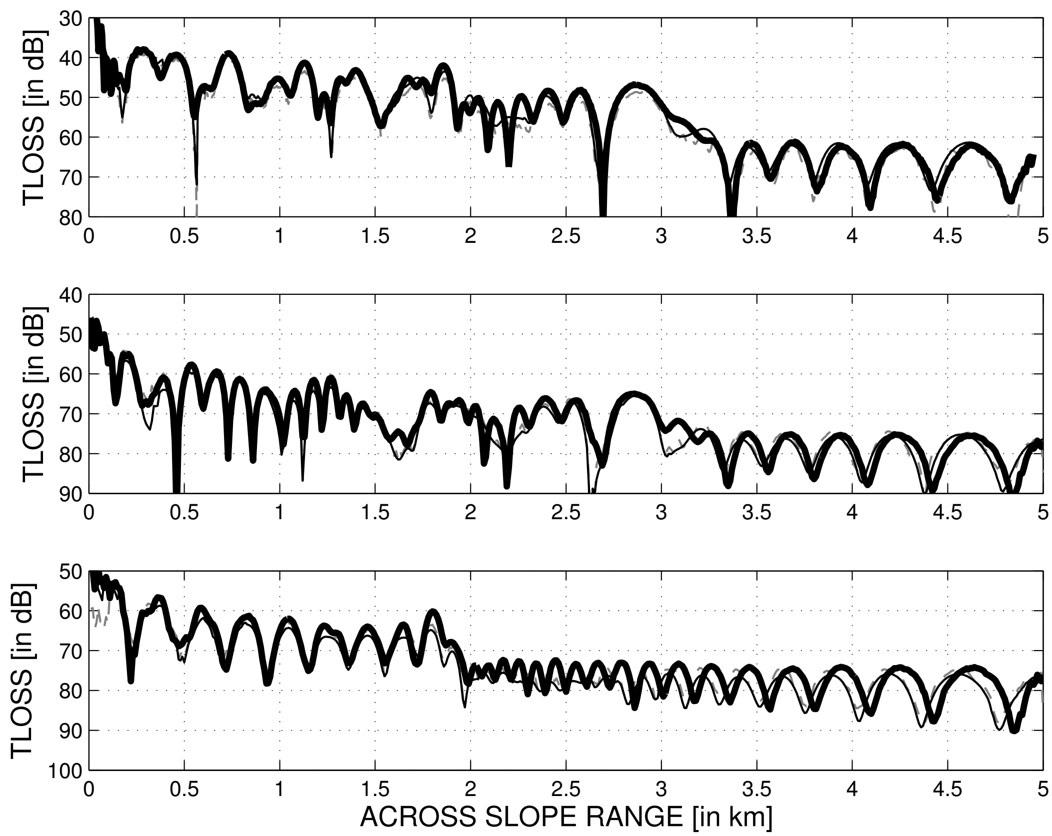

Figure 5: Benchmarking results for experimental data and different positions of the acoustic source: H1 (top), H2 (middle) amd H3 (bottom); color convention: black, reference; thin black, Bellhop3D; dashed gray, TRACEO3D.

be the fastest model in terms of computational speed, followed by KRAKEN and TRACEO3D; certainly, the current version of TRACEO3D will be improved to allow faster computations. The specific numerical strategies adopted by each model are worth additional discussion, and will be addressed in detail in the future. Development of studies for real environments, with complex bathymetries and sound speed fields, will be also of fundamental importance.

ACKNOWLEDGMENTS

The authors are deeply thankful to LMA-CNRS for allowing the use of the experimental tank data discussed in this work.

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