MODEL-BASED CORRELATORS: INTERESTING CASES IN UNDERWATER ACOUSTICS

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ABSTRACT

Model-based correlators are in nowdays widely used in disciplines such as communications and underwater acoustics. These class of methods generally make strong assumptions on the data underlying physical model and develop dedicated processors that fully exploit the model structure. When the assumed model is a good description of the data, these methods achieve very high quality results since, in the mismatch free case, they are optimal for the single signal in white noise scenario. However, in presence of model mismatch this result is degraded. This paper presents a theoretical study that addresses the optimality of model-based correlators when compared with straight correlators in a model mismatch situation. It is shown that model-based correlators can sufer severe degradation in relatively mild mismatch scenarios and - more importantly - in that case model-based correlators are outperformed by simple signal correlators. The theory is supported by a few realistic examples drawn from underwater acoustics.

1. INTRODUCTION

In the early days of signal processing, reasoning and methodologies were essentially information based. Most of the methods were based on assumed statistical properties of the data, like for instance, signal coherence, decorrelation between signal and noise, noise distribution, etc...These methods were generally called as 'data driven' methods. In order to give an example, let's say that the signal x(t) was observed through the equation

$$y(t) = x(t) + n(t) \tag{1}$$

where n(t) is an additive noise. At this point one would normally make use of a priori knowledge to assume whether the signal x(t) is deterministic or stochastic, broadband or narrowband and, in case, assume its bandwidth and the usual statistical properties on the noise process like uncorrelation with the signal and eventually its distribution. These assumptions were suficient to derive methods for estimation, filtering or detection depending on the objective sought. The work done by well known names like Wiener [1], Shannon [2] and Bartlett [3], to cite only a few, in time series analysis are in nowadays part of most text books. In the 60s a new idea, based on the parametric representation of the signal part of the data model, was introduced. The basics of this approach steam from the representation of the signal as a generic model depending on a number of abstract parameters. Well known and widely used models are for example auto-regressive (AR), auto-regressive moving-average (ARMA) and weighted sum of exponentials. The problem of signal estimation is transformed into the problem of estimating the parameters of the parametric model. The works of Capon [5] and Burg [6] in spectrum estimation and that of Kalman and Bucy in optimal estimation from time series are well known. The underlying ideia is that those methods would have a superior performance than previous plain time series analysis by restricting the signal search space to that space defined by a given model structure. In other words, intuitively is easier to search for a signal belonging to a class representable by a given model than for a signal of any kind.

In this context, the methods based on subspace decomposition - noise and signal subspace -play a mixed role, since they allow for restricting the search space by confining the signal to a reduced class, without explicitely introducing any parametric model. We would consider these methods in the class of 'data-driven' methods even if they only appeared in the 70s with the parallel work of Bienvenu [7] and Schmidt [8], without forgetting the percursor work of Prony [9] and Pisarenko[10]. More recently, in the 80s, and possibly due to the easy access to computers with high computing power, appeared the methods based on physical models of the signal. Thus, with these models, the signal is no longer described by parametric models with abstract parameters but instead they can be directly linked to models with parameters having a physical meaning. Signal x(t) in equation 1 becomes the output of a system represented by a set of differential equations that describe the modifications of the input signal between the emitting location and the point of observation. Finding this set of differential equations generally depends on a process of numerical integration of some kind of wave equation in the media of propagation with the respective boundary conditions. These are the so called 'model-based' methods. Compared with the parametric models the model-based methods are more restrictive and therefore supposed to yield, in principle, better results. Another reason why the 'model-based' methods are supposed to yield better results is that they include a higher amount of a priori information. The question that is the focus of this paper is to determine whether and, in case, in which conditions do the 'model-based' methods perform better than classical methods. This question will be answered first with a theoretical study and then with some simulated examples drawn from realistic cases in shallow water underwater acoustics.

2. THEORETICAL BACKGROUND

Equation 1 is a representation of the observation of signal x(t) in presence of an additive noise n(t). The inclusion of the physical model concept steams from the idea of considering that x(t) is the output of an linear system, thus one can write

$$x(t; r) = g(t; r) * s(t) \qquad (2)$$

where g(t, r) is the impulse response of the system characterized, in this case, by the ensemble of parameters jointly represented under the form of vector r, and where s(t) is the emitted signal or the system input. There are a number of hypothesis that can be made regarding the quantities in 2. First, the input signal s(t) maybe considered either deterministic or stochastic and in the later it can be considered as correlated or uncorrelated with the observation noise n(t) of 1. Similarly, even if the system function g(t; r) is generally assumed as deterministic it may depend on a parameter vector r that itself maybe deterministic or stochastic with a known or partially known or even completely unknown statistical distribution. In general, whether depending on s(t) or r, the signal x(t) is considered of random nature. Based on practical grounds it makes in general sense to consider that the signal x(t) and the noise n(t) to be statistically uncorrelated.

Let us point out that the fact of considering a physical dependence for x(t), not only may allow for a better estimation/detection of the signal x(t) itself, but also opens up the possibility for identifying physical parameters of the environment of propagation between the emitter and the receiver. That fact, completely changes the interest of the approach in other fields such as ocean and biomedical tomography, environmental monitoring, geoacoustic exploration, etc...Therefore 'model-based' methods may lead to one of two main objectives: estimating the source signal - generally called deconvolution - that we will not follow here, or identifying g(t; r) or r also termed as channel identification. This double problem may be analized through a unique approach from a generalized notion of matched-filter (MF) that in that case takes the name of generalized matched-filter (GMF).

2.1. The generalized matched-filter

Let

$$x(t; r) = g(t; r) *s(t); \qquad (3)$$

and

$$y(t; r) = x(t; r) + n(t); \qquad (4)$$

be the model equations for the signal and the observation, respectively. In this model, onewants to estimate x(t; r) conditioned in the function g(t; .) and in the parameter vector.

This is a classical problem in signal theory, which most celebrated solution is given by the Wiener filter [1], that minimizes the mean square error between the filtered signal plus noise and the expected signal. That is equivalent to the minimization of the noise power at the output of the filter or even - equivalentely - the maximization of the signal to noise ratio (SNR). Such a filter is well known to be the matched-filter. Its derivation is shown in almost every textbook in signal theory. We will follow here the approximation of Davenport [4] with the generalization that takes into account the dependence on the parameter vector **r**.

The problem can be stated as to determine the filter h(t; **r**) given by

$$z(t,\mathrm{r}) = h(t,\mathrm{r}) * y(t,\mathrm{r}) = z_o(t,\mathrm{r}) + n_o(t), \qquad (5)$$

such that the ratio between the signal energy and the mean energy of the noise (SNR), defined as

$$\rho(t,\mathrm{r}) = \frac{|z_o(t,\mathrm{r})|^2}{< n_o^2 >}, \qquad (6)$$

is maximum. Replacing 3 and 4 in 5 gives

$$z_o(t,\mathrm{r}) = \frac{1}{2\pi}\int_{-\infty}^{\infty} H(\omega,\mathrm{r})G(\omega,\mathrm{r})S(\omega)e^{j\omega t}d\omega, \qquad (7)$$

where H(ω; r), G(ω; r) and S(ω) are the Fourier Transforms (FT) of h(t; r), g(t; r) and the signal s(t) respectively. We can also write, using some basic properties of the correlation function, that the mean energy of the noise at the filter output is

$$< n_o^2 >= \phi_{n_o n_o}(\tau)|_{\tau=0} = \frac{1}{2\pi}\int_{-\infty}^{\infty} |H(\omega,\mathrm{r})|^2 P_{nn}(\omega)d\omega, \qquad (8)$$

where $\phi_{n_o n_o}(\tau)$ is the autocorrelation function of the filtered noise $n_o(t)$ and $P_{nn}(\omega)$ is the power spectral density of the noise n(t) at the input of the filter. Replacing now 7 and 8 in 6 and reducing the integration interval to the portion of the spectra of interest $\omega \in \Omega$, gives

$$\rho(t,\mathrm{r}) = \frac{1}{2\pi}\frac{\left|\int_\Omega H(\omega,\mathrm{r})G(\omega,\mathrm{r})S(\omega)e^{j\omega t}d\omega\right|^2}{\int_\Omega |H(\omega,\mathrm{r})|^2 P_{nn}(\omega)d\omega}. \qquad (9)$$

It can be shown (appendix A) that the maximum of the output SNR, equation 9, is attained. When

$$H(\omega, \mathbf{r}) = H_0 \frac{G^*(\omega, \mathbf{r}) S^*(\omega, \mathbf{r})}{P_{nn}(\omega)} e^{-j\omega\tau}, \tag{10}$$

where H_0 is a constant of proportionality and τ represents an arbitrary phase shift. The maximum attainable SNR is obviously given by replacing 10 in 9 and by searching for the value of t that maximizes the correlation on the numerator, giving

$$\rho_{\max}(\mathbf{r}) = \frac{1}{2\pi} \int_\Omega \frac{|G(\omega, \mathbf{r})|^2 |S(\omega)|^2}{P_{nn}(\omega)} d\omega. \tag{11}$$

Starting from the expression of the generalized matched-filter (GMF), equation 10, a number of estimators can be derived depending if one wants to determine the channel response g(t;r), or the parameter vector r, or the emitted signal s(t), conditioning each case in the a priori knowledge of the others. In all cases it is assumed that the power spectral density of the noise $P_{nn}(\omega)$, is known or can be estimated. In practice the observation noise is often considered to be white and of zero mean, which, as will be seen in the sequel, will signi_cantly simplify the expressions of the performance of the GMF.

2.2. Performance of the generalized matched-fillter

In order to allow a simpler analysis of the performance of the GMF, the SNR relation 9 may be written as a function of the correlation product and its maximum attainable value

$$\rho(t, \mathbf{r}) = \frac{1}{2\pi} \frac{\left| \int_\Omega \frac{\hat{G}^*(\omega, \mathbf{r}) G(\omega, \mathbf{r}) |S(\omega)|^2}{P_{nn}(\omega)} e^{j\omega(t-\tau)} d\omega \right|^2}{\int_\Omega \frac{|\hat{G}(\omega, \mathbf{r})|^2 |S(\omega)|^2}{P_{nn}(\omega)} d\omega}, \tag{12}$$

where the filter H(ω; r) was replaced by its estimate $\hat{H}$ (ω; r),

$$\hat{H}(\omega, \mathbf{r}) = \frac{\hat{G}^*(\omega, \mathbf{r}) S^*(\omega, \mathbf{r})}{P_{nn}(\omega)} e^{-j\omega\tau}. \tag{13}$$

In this expression the estimator $\hat{H}$, of *H*, was introduced as dependent on the estimator $\hat{G}$ of G conditioned in vector r. However, we could as well have written $\hat{H}$ as a function of the model G(ω; $\vec{r}$), now as a function of the estimator $\hat{r}$ of r, conditioned on function G,

$$\hat{\mathbf{r}} = \arg\max_{\mathbf{r}} \left[\max_t \rho(t, \mathbf{r}) \right]. \tag{14}$$

In other words, equation 13 can be viewed as an estimator, maximizing SNR (or minimizing the mean least square error [1]), of r as well as of G, depending on the problem at hand. Multiplying and dividing 12 by ρ_{max}(r) we get

$$\rho(t,\mathrm{r}) = \rho_{\max}(\mathrm{r})|\Lambda(t,\mathrm{r})|^2, \tag{15}$$

with

$$\Lambda(t,\mathrm{r}) = \frac{\displaystyle\int_\Omega \frac{\hat{G}^*(\omega,\mathrm{r})G(\omega,\mathrm{r})|S(\omega)|^2}{P_{nn}(\omega)} e^{j\omega(t-\tau)}d\omega}{\left[\displaystyle\int_\Omega \frac{|\hat{G}(\omega,\mathrm{r})|^2|S(\omega)|^2}{P_{nn}(\omega)}d\omega\right]^{1/2}\left[\displaystyle\int_\Omega \frac{|G(\omega,\mathrm{r})|^2|S(\omega)|^2}{P_{nn}(\omega)}d\omega\right]^{1/2}}, \tag{16}$$

that is nothing else than the normalized ratio of the correlation between the estimator $\hat{G}$ (ω; r) and its true value G(ω; r). In that case we have that $0 \leq \Lambda(t; r) \leq 1$. In order to make it clear let us present a few examples and some particular and common cases.

2.2.1 The white noise case

In the case that the observation noise allows for a correlation function of the type

$$\phi_{nn}(\tau) = \frac{N_0}{2}\delta(\tau), \tag{17}$$

we have $P_{nn}(\omega) = N_0/2$ and the maximum of the attainable SNR is

$$\rho_{\max}(\mathrm{r}) = \frac{2\epsilon_x}{N_0}, \tag{18}$$

where c_x is the energy of the signal x(t) given by

$$\epsilon_x = \frac{1}{\pi}\int_0^\infty |X(\omega,\mathrm{r})|^2 d\omega. \tag{19}$$

In this simple case the ratio Λ is

$$\Lambda(t,\mathrm{r}) = \frac{\displaystyle\int_\Omega \hat{G}^*(\omega,\mathrm{r})G(\omega,\mathrm{r})|S(\omega)|^2 e^{j\omega(t-\tau)}d\omega}{\left[\displaystyle\int_\Omega |\hat{G}(\omega,\mathrm{r})|^2|S(\omega)|^2 d\omega\right]^{1/2}\left[\displaystyle\int_\Omega |G(\omega,\mathrm{r})|^2|S(\omega)|^2 d\omega\right]^{1/2}}. \tag{20}$$

without loss of generality the observation noise will be considered white in the sequel.

2.2.2 Signal with a flat spectrum

Another case that often arises in practice is when the emitted signal s(t) has a flat spectrum within the bandwidth of interest, i.e.,

$$S(\omega) = S_0, \qquad \forall \omega \in \Omega. \tag{21}$$

In this case we have that

$$\Lambda(t, \mathbf{r}) = \frac{\int_\Omega \hat{G}^*(\omega, \mathbf{r}) G(\omega, \mathbf{r}) e^{j\omega(t-\tau)} d\omega}{\left[\int_\Omega |\hat{G}(\omega, \mathbf{r})|^2 d\omega\right]^{1/2} \left[\int_\Omega |G(\omega, \mathbf{r})|^2 d\omega\right]^{1/2}}. \tag{22}$$

2.2.3 Optimal filter

The optimal filter is obtained when the channel is completely known or can be perfectly estimated. In that case

$$\hat{G}(\omega, \mathbf{r}) = G(\omega, \mathbf{r}), \tag{23}$$

and therefore Λ(t; r) = 1 and

$$\rho_{\mathrm{GMF}}(\mathbf{r}) = \rho_{\max}(\mathbf{r}). \tag{24}$$

Therefore we can say that the optimal filter is the one that reaches the maximum attainable output SNR, and that filter is the GMF or the model-based matched filter. Note that assumption 23 (inserted in 16 or 20) does not imply equality for all values of ω but only that the impulse response of the channel should be known up to an arbitrary coeficient (≠ 0) in amplitude H_0 and a phase delay τ

2.3. Matched-filter (MF)

On the other hand if the channel impulse response is completely unknown we have that $\hat{G}(\omega, \mathbf{r}) = 1 \ \forall \omega \in \Omega$, and in that case,

$$\rho_{\mathrm{MF}}(\mathbf{r}) = \rho_{\max}(\mathbf{r}) \alpha(\mathbf{r}), \tag{25}$$

where α(r) represents the maximum of the correlation function Λ when $\hat{G}$ (ω; r) = 1,

$$\alpha(\mathbf{r}) = \frac{\max_t \left| \int_\Omega G^*(\omega, \mathbf{r}) |S(\omega)|^2 e^{j\omega(t-\tau)} d\omega \right|^2}{\int_\Omega |S(\omega)|^2 d\omega \int_\Omega |G(\omega, \mathbf{r})|^2 |S(\omega)|^2 d\omega.} \quad (26)$$

In this case we obtain the well known conventional matched-filter or simply matche-filter (MF), that has only into account the emitted signal s(t) and not the impulse response of the channel g(tr), i.e.,

$$H(\omega, \mathbf{r}) = H_0 S^*(\omega) e^{-j\omega\tau}. \quad (27)$$

2.4. Relative performance GMF vs MF

In the more frequent intermediate case when the channel impulse response is partially known, the degree of similarity between the estimated impulse response and the true impulse response is given by

$$0 \leq \max_t |\Lambda(t, \mathbf{r})|^2 \leq 1.$$

On the other hand, the reduction of the output SNR is due to the time spreading of the signal energy and can be quantified by the value of $\alpha(\mathrm{r}) \leq 1$. Thus, the gain oferred by the model-based processor relative to the matched-filter can be quantified by the ratio

$$\frac{\rho_{\mathrm{MF}}(\mathbf{r})}{\rho_{\mathrm{GMF}}(\mathbf{r})} = \frac{\max_t |\Lambda(t, \mathbf{r})|^2}{\alpha(\mathbf{r})}. \quad (28)$$

3. SIMULATION RESULTS

A typical area of application of estimation theory is underwater acoustic signal processing. This section gives a number of examples drawn from realistic cases to illustrate the theoretical assessment made in the previous section. Let us consider the case of a multitone signal with 50 frequencies between 50 and 150 Hz every 2 Hz. The signal is transmitted in a 135 m depth waveguide with a slightly downward refracting sound speed profile (table I) over a sandy bottom characterized by a 1750 m/s sound speed, a density of 1.9 g/cm3 and a compressional attenuation of 0.8 dB/λ, (figure 1).

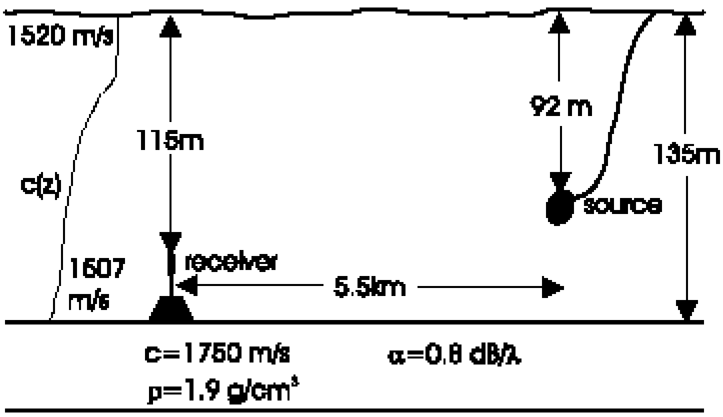

Figure 1. *Environmental scenario used for simulation*

The source was located at 92 m depth and the receiver was located at 115 m depth. Source-receiver range was variable between 0 and 5 km. Figure 2 shows the results obtained for ratios Λ (eq.16), α (eq.26) and GMF/MF performance (eq.28) in case of a water depth mismatch of 1 m. This means that the difference between G and $\hat{G}$ was created by an environmental diference of water depth of 1/135 < 1%. What can be observed in this figure is that the normalized correlation ratio is close to 1 for ranges up to, say 1 km, and then slowly decreases for larger distances. Signal dispersion coeficient α strongly oscillates at all ranges. The net result of the ratio between the two gives a gain curve that is well above one for ranges ≤ 2 km. For ranges between 2 and 5 km, the gain of GMF over MF is not always above one and sometimes is well below for some quite large range intervals.

Depth (m)	Sound speed (m/s)
0.0	1520
5.0	1520
11.3	1518
21.3	1516
32.0	1512
42.7	1510
72.8	1508
94.6	1507
135.0	1507

Table 1. Sound speed profile used in the simulartion example

The next test is an extension of the precedent one for a two dimensional parameter with source range and depth, r = (z; r). Therefore Λ, α and the GMF/MF gain are now ambiguity surfaces showed in figure 3. It can be observed that the variation with depth is small compared to the previous case, i.e., high correlation at short range with some gain of the GMF over the MF processor while after, say 2.5 km, the gain actually drops well below one with the conventional MF processor performing better than the GMF.

In a third example, we address a different case which is geoacoustic inversion problem: we want to estimate, for example, the compressional velocity csec in the sediment layer, while knowing the source position with a small error in depth of 5m. The three ambiguity surfaces are shown in figure 4 for Λ, α and GMF/MF gain as a function of sediment C-speed versus source range. Here the behaviour is quite different since decorrelation does not nearly monotonically decrease with source-receiver range but oscillates beetween low and high values, even at long ranges, giving rise to some high GMF/MF gains (up to 8) at ranges of 4.5 km.

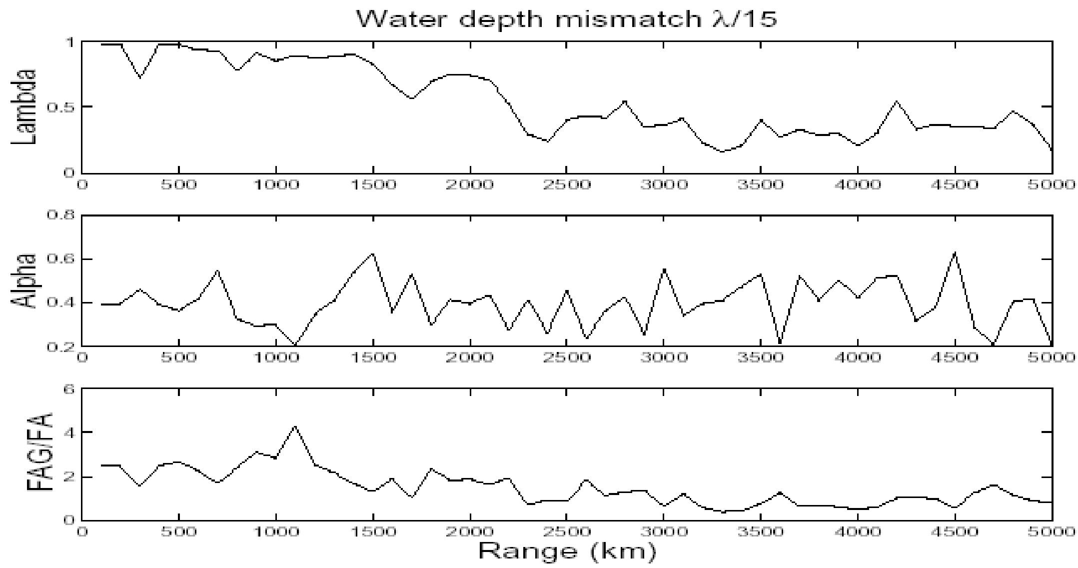

Figure 2. *Water depth mismatch of 1m: quantities Λ, α and GMF/MF ratio versus source-receiver range*

4. CONCLUSION

Easy and inexpensive access to computational means have lead, in the last 10 - 15 years, to a ever increasing usage of physical models in signal processing. These methods - so called model-based methods - make fully coherent use of a priori information about the physical process andenvironment of signal transmission to retrieve the maximum amount of information about boththe emitting source and the transmission channel.

Model-based methods can be very rewarding since they allow for theoretically perfect matched-filter implementations when the physical model perfectly fits the real channel. Also, when the channel of propagation is not known but the source signal is under experimental control the process can be reversed to estimate the channel of propagation and identify the physical parameters, with clear advantages in many applications. What has been observed in practice is that even if model-based methods have been in "the market" for over 20 years now, its acceptance in real-world applications has been slow. The reason for that is known: model-based methods are superior to simple data driven methods only if the information fed in the models is accurate enough. In this paper we have attempted to quantify and theoretically explain where does that lack of superiority comes from and also give some practical simulated examples drawn from underwater acoustics to illustrate the comparative methods in play.

The bottom line is that conventional matched-filtering (without any knowledge about the environment) can have higher SNR output than generalized matched filtering depending on signal time spread at the receiver and on the degree of mismatch on the environmental characteristics. The simulated examples show that for relatively mild - and fairly realistic – degrees of mismatch the gain provided by the model-based methods over the conventional methods can drop well below one in practical situations both of source localization and geoacoustic inversion - which have been two areas of intense activity and usage of *model based* methods in the last years.

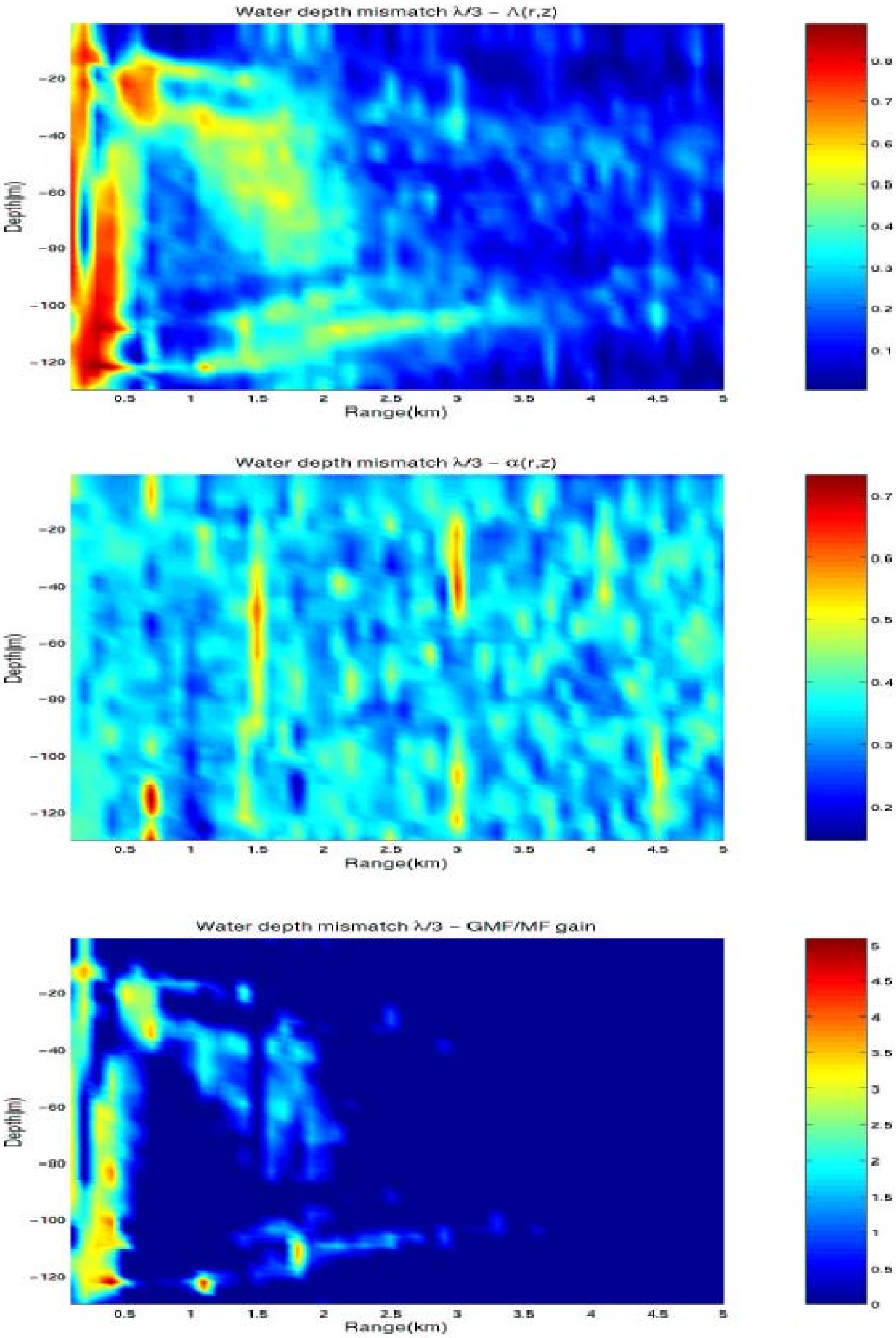

Figure 3. *Water depth mismatch of 5m: quantities Λ, α and GMF/MF ratio versus source range and depth*

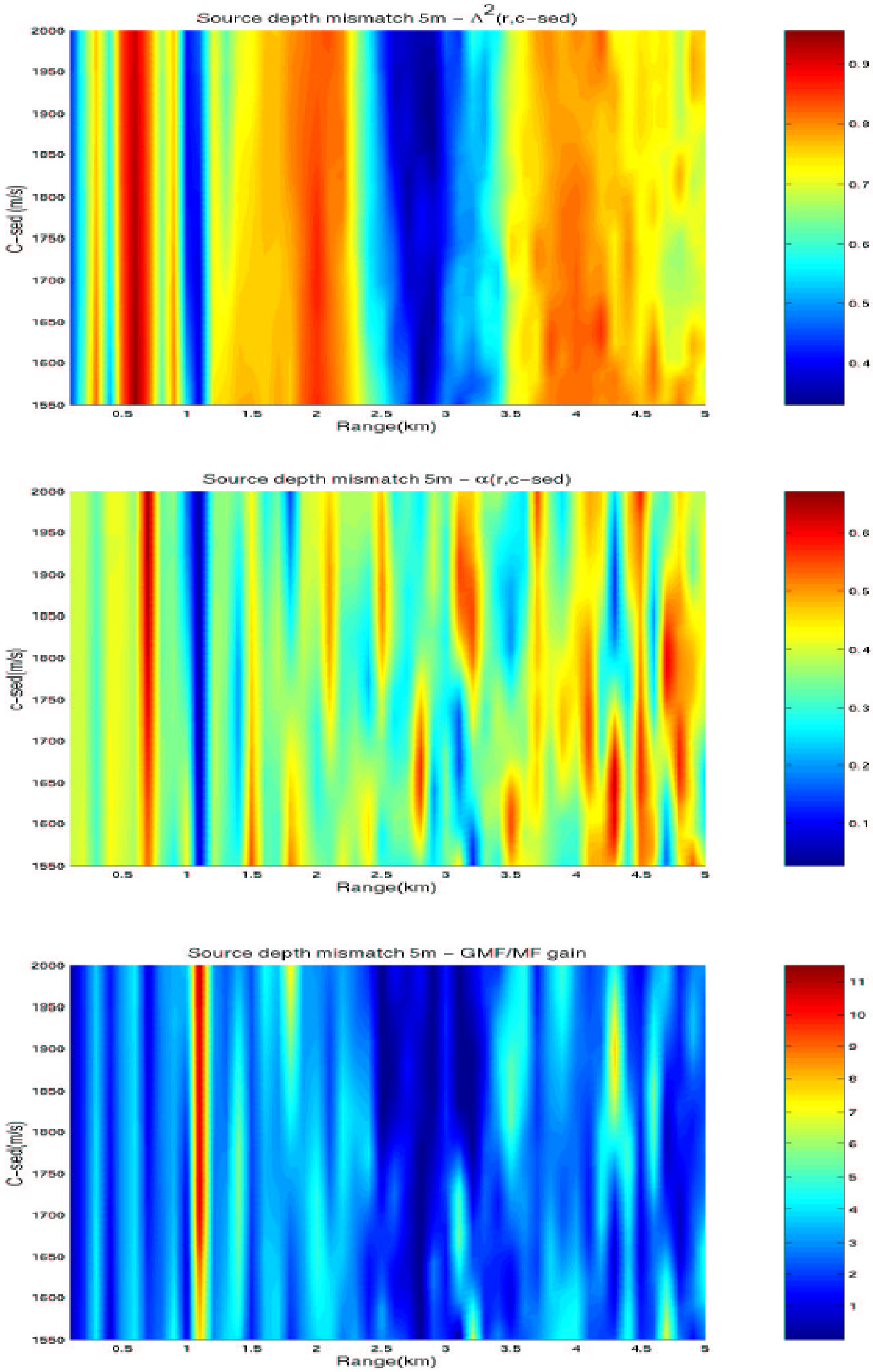

Figure 4. *Source depth mismatch of 5m: quantities Λ, α and GMF/MF ratio versus sediment com pressional velocity and source-receiver range*

5. APPENDIX

Matched-filter derivation

Starting from equation 9 and assuming that the power spectral density of the noise is different from zero in the considered frequency band, one can multiply and divide the numerator by $\sqrt{P_{nn}(\omega)}$, giving

$$\rho(t,\mathrm{r}) = \frac{1}{2\pi}\frac{\left|\int_{-\infty}^{\infty}[G(\omega,\mathrm{r})S(\omega)/\sqrt{P_{nn}(\omega)}][H(\omega,\mathrm{r})\sqrt{P_{nn}(\omega)}e^{j\omega\tau}d\omega]\right|^2}{\int_{-\infty}^{\infty}|H(\omega,\mathrm{r})|^2 P_{nn}(\omega)d\omega}. \quad (29)$$

On the other hand, from the Schwartz inequality, one can say that

$$|\int f(x)g(x)dx|^2 \leq \int |f(x)|^2 dx \int |g(x)|^2 dx, \quad (30)$$

and write for 29,

$$\rho(t,\mathrm{r}) \leq \frac{1}{2\pi}\frac{\int_{-\infty}^{\infty}|G(\omega,\mathrm{r})S(\omega)/\sqrt{P_{nn}(\omega)}|^2 d\omega \int_{-\infty}^{\infty}|H(\omega,\mathrm{r})\sqrt{P_{nn}(\omega)}e^{j\omega\tau}|^2 d\omega}{\int_{-\infty}^{\infty}|H(\omega,\mathrm{r})|^2 P_{nn}(\omega)d\omega}. \quad (31)$$

From 30 one can ee that the identity is only reached when f(x) = g(x) or that, for our case, when

$$\frac{G(\omega,\mathrm{r})S(\omega,\mathrm{r})}{\sqrt{P_{nn}(\omega)}} = [H(\omega,\mathrm{r})\sqrt{P_{nn}(\omega)}e^{j\omega\tau}]^*, \quad (32)$$

and that in this case simplifying 31 by

$$\int_{-\infty}^{\infty}|H(\omega,\mathrm{r})|^2 P_{nn}(\omega)d\omega, \quad (33)$$

one obtains the maximum value of ρ that is

$$\rho_{\max}(\mathrm{r}) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{|G(\omega,\mathrm{r})|^2|S(\omega)|^2}{P_{nn}(\omega)}d\omega. \quad (34)$$

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