# DEPTH AND RANGE SHIFT COMPENSATION USING WAVEGUIDE INVARIANT PROPERTIES

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Abstract: In most underwater acoustic experiments acoustic sources and hydrophone arrays are moored so as to provide a geometry as controllable as possible. A more operational approach is to use moving sources and light drifting acoustic buoys but in such cases the data exhibits continuous phase and amplitude changes due to depth and range shifts. This may be problematic when the processing of collected acoustic data requires the use of correlation between successive received signals, e.g., in passive time reversal where a probe-signal is sent ahead of the data-signal for post crosscorrelation. An identical problem arises when the source is placed in a continuously moving and unstable autonomous underwater vehicle. Up to now, only the range shift is usually compensated by using data processing techniques, for example, by applying an appropriate frequency shift to the acoustic field based on waveguide invariant theory. This paper formulates the hypothesis that array and source depth fluctuations can also be compensated by using a frequency shift of the received pressure field. Acoustic simulations and real data collected during the MREA'04 experiment suggest that the frequency translation required for the array and source depth compensation can be computed using new invariant properties of the waveguide.

*Keywords:* Acoustic time reversal, waveguide invariant, shallow water propagation, normal mode models.

# **1. INTRODUCTION**

A realistic scenario for Rapid Environment Assessment (REA) with acoustic means involves an Autonomous Underwater Vehicle (AUV) performing an underwater survey and an easy to deploy free drifting buoy as interface between the AUV and the operational command [1]. In that context an acoustic communication link between the AUV and the free drifting buoy is required and it would be important to have a real time geometric characterization of the scenario, including source-array range and array and source depths. Passive time reversal (pTR), because of its simplified equipment and processing setup, can be used for acoustic communication between the AUV and the surface free drifting buoy. In fact, pTR only requires the use of a source and single receiving array and a probe-signal should be transmitted ahead of the data-signal in order to provide an estimate of underwater channel Impulse Responses (IRs). Time focusing is then performed synthetically at the array side by simply convolving a TR version of the estimated IRs with the incoming data-signal. In the presence of a moving source and a free drifting array pTR rapidly looses its time focusing capability mainly due to geometric mismatch, and up to now no attempt has been made to incorporate the geometric tracking capability in pTR.

Active Time Reversal (aTR) is a conceptually similar technique where previous work by Song el al. [2] addresses the focal spot range shift problem by using a frequency translation of the array received acoustic field. The technique can be readily applied in pTR, making it possible to perform source-array range tracking. In [3] a different strategy is presented for aTR focal spot depth shift, but it is not versatile for a real time automatic application.

This paper presents a set of simulations and real data observations that support the hypothesis that frequency shifting the probe-signal not only compensate for range shift between the probe-signal and the data-signal transmissions, but it can also be used to compensate for the source and the receiving array depth shifts. If applicable to pTR-based underwater communications, such techniques will allow for the dynamic compensation of the geometric mismatch along time between the probe-signal and the data-signal transmissions.

#### 2. FREQUENCY SHIFT COMPENSATION OF GEOMETRIC MISMATCH

In the frequency domain at ranges greater than a few water depths the acoustic field generated by a monochromatic point source at each hydrophone of a receiving array with *I* hydrophones can be given by its normal modes *Green's* function

$$H_{i}(\omega) = \frac{je^{-j\frac{\pi}{4}}}{\rho\sqrt{8\pi R}} \sum_{m=1}^{M} \Psi_{m}(\varsigma_{0})\Psi_{m}(\varsigma_{i}) \frac{e^{jk_{m}(\omega)R}}{\sqrt{k_{m}(\omega)}}$$
(1)

where *i* is the hydrophone index, *m* is the mode number,  $\rho$  is the (assumed) constant water density, *R* is the source array range,  $\psi_m$  is the *m* mode shape,  $k_m$  is the mode horizontal wavenumber,  $\varsigma_0$  is the source depth,  $\varsigma_i$  is the hydrophone depth, and  $\omega$  is the signal frequency. The acoustic field synthetically generated by pTR in a static environment is given by

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$$P_{PC;\omega,R,\varsigma_{i},\varsigma_{0}} = \frac{1}{\rho^{2} 8\pi R} \sum_{n=1}^{M} \sum_{m=1}^{M} \frac{\Psi_{m}(\varsigma_{0})\Psi_{n}(\varsigma_{0})}{\sqrt{k_{m}(\omega)}\sqrt{k_{n}(\omega)}} \sum_{i=1}^{I} \Psi_{m}(\varsigma_{i})\Psi_{n}(\varsigma_{i})e^{jk_{m}(\omega)R}e^{-jk_{n}(\omega)R}$$

$$= \frac{1}{\rho^{2} 8\pi R} \sum_{n=1}^{M} \frac{|\Psi_{m}(\varsigma_{0})|^{2}}{|k_{m}(\omega)|}$$
(2)

where, for simplicity, it is considered that  $\Psi_m(\cdot)$  and  $k_m(\cdot)$  are real quantities, which is similar to ignore the leaky modes and consider the loss mechanisms to be negligible. Additionally, in (2) it was considered that the array spans the entire water column and that hydrophone spacing is sufficiently dense in order to satisfy the orthogonality of sampled normal modes. Since  $P_{PC}$  is approximately constant in the frequency domain, in the time domain it will result in a weighted dirac  $P_{TR}\delta(t)$ . The notation  $P_{TR}$  will be used in the illustrative examples and  $P_{PC}$  will be used in the mathematic formulation.

In the following it will be considered that the pTR is applied with two consecutive probe-signals. The mismatch in range, array depth and source depth affects the second transmitted probe-signal (index *n* in (2)), and compensation is applied to the first transmitted probe-signal (index *m* in (2)). The resulting ambiguity surface  $P_{TR;R,\varsigma_i,\varsigma_0}$  (*mismatch, compensation*) will be observed in the time domain. The simulations were performed with the C-SNAP normal mode model in a range independent environment, with 120 m water column depth, over a 1.5 m silt bottom and gravel subbottom. The sound speed profile is downward refracting, varying between 1512 and 1505 m/s with a thermocline in the first 15 m. The canonical geometric values are 70 m source depth, 700 m source-array range, 32 uniformly-spaced array elements with a 60 m aperture such that the last hydrophone is placed at 89 m, and the source-array range is 700 m. Probe-signals are narrowband pulses from 3.2 to 4 kHz.

#### 2.1 Range Compensation

In the presence of range mismatch between two consecutive probes the resulting pTR acoustic field, compensated by a frequency shift according to [2], will be given by

$$P_{PC;\cdot}(\Delta r, \Delta \omega) = \frac{1}{\rho^2 8\pi R} \sum_{n=1}^{M} \sum_{m=1}^{M} \frac{\Psi_m(\varsigma_0)\Psi_n(\varsigma_0)}{\sqrt{k_m(\omega)k_n(\omega)}} \sum_{i=1}^{I} \Psi_m(\varsigma_i)\Psi_n(\varsigma_i) \times e^{jk_m(\omega+\Delta\omega)R} e^{-jk_n(\omega)(R+\Delta r)}$$
(3)

where in the denominator the approximations  $\sqrt{k_m(\omega + \Delta \omega)k_n(\omega)} \approx \sqrt{k_m(\omega)k_n(\omega)}$  and  $R \approx \sqrt{R(R + \Delta r)}$  were used. In (3) it is clear that the range shift  $\Delta r$  mainly affects the exponential term, and the compensation mechanism will use a frequency shift  $\Delta \omega$  in order to make the approximate  $e^{jk_m(\omega + \Delta \omega)R} \approx e^{-jk_n(\omega)(R + \Delta r)}$  for m = n. Fig 1 shows the resulting ambiguity surface  $P_{TR:}(\Delta r, \Delta \omega)$ , where the dashed line represents the non compensated case, \* marks the empirical maximum for each range shift and the continuous line traces the maxima predicted by waveguide invariant theory. Fig. 1(a) shows  $|P_{TR:}(\Delta r, \Delta \omega)|$ , where by comparing the dashed line with the other two one can see

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that up to a range shift of 45 m there is a gain due to the compensation. Fig. 1(b) shows the  $P_{TR;}(\Delta r, \Delta \omega)$  unwraped phase in the previously defined trajectories, where one can see that the linear approximation results in a linear variation of phase with the range shift, which is of main importance for coherent communications since it simplifies phase tracking.



Fig.1: Simulated  $P_{TR:}(\Delta r, \Delta \omega)$  ambiguity surface, magnitude (a), angle (b)

Such compensation is made possible by the waveguide invariant property [2] by using the slope invariant  $\beta$  that can be considered approximately constant for a limited group of modes and narrowband signals. In the present simulation the number of propagating modes is limited to 500.

#### 2.2 Array Depth and Source Depth Compensation

In the presence of array and source depth mismatches between consecutive probes, the resulting frequency shift compensated pTR acoustic field will be given by

$$P_{PC;\cdot}(\Delta\varsigma_A, \Delta\varsigma_S; \Delta\omega) = \frac{1}{\rho^2 8\pi R} \sum_{n=1}^{M} \sum_{m=1}^{M} \frac{\Psi_m(\varsigma_0)\Psi_n(\varsigma_0 + \Delta\varsigma_S)}{\sqrt{k_m(\omega)k_n(\omega)}}$$

$$\times \sum_{i=1}^{I} \Psi_m(\varsigma_i)\Psi_n(\varsigma_i + \Delta\varsigma_A) e^{jk_m(\omega + \Delta\omega)R} e^{-jk_n(\omega)R}$$
(4)

where  $\Delta \varsigma_A$  is the array depth shift,  $\Delta \varsigma_S$  is the source depth shift, and  $\Delta \omega$  is the frequency translation compensation mechanism. In (4) there exists a violation of mode orthogonality due to  $\Delta \varsigma_A$  and  $\Delta \varsigma_S$  that causes a loss in performance due to the out-of-phase product of the two source position mode shapes. In both cases the compensation is made by a frequency shift  $\Delta \omega$  over  $k_m$ . Fig. 2 shows the resulting  $P_{TR}(\cdot)$  ambiguity surface for the compensated array depth mismatch (Figs. 2a-b) with  $\Delta \varsigma_S = 0$ , and for the compensated source depth mismatch (Figs. 2c-d) with  $\Delta \varsigma_A = 0$ , where the behaviour is similar to the one observed in Fig. 1 and similar comments apply. Such similarity allows for the intuitive idea that here there should be a waveguide invariant similar to  $\beta$  in Sec. 2.1.



*Fig.2: Simulated*  $P_{TR;\cdot}(\Delta \varsigma_A, \Delta \omega)$  *ambiguity surface, magnitude (a), angle (b); Simulated*  $P_{TR;\cdot}(\Delta \varsigma_S, \Delta \omega)$  *ambiguity surface, magnitude (c), angle (d)* 



Fig.3: Real data  $P_{TR;.}(\Delta r, \Delta \varsigma_A; \Delta \omega)$  ambiguity surface, magnitude (a), maximum trajectory power spectrum density (b)

The real data were collected during the MREA'04 sea trial [4], with the source suspended at a nominal depth of 70 m, from the NRV Alliance in free drifting mode, and an 8-hydrophone array suspended from the also free drifting Acoustic Oceanographic Buoy [1]. Under such conditions only range mismatch  $\Delta r$  and array depth mismatch  $\Delta \varsigma_A$ 

are expected for each  $\Delta t$  between the initial and subsequent IRs estimated from an 18second PSK data communication signal after Doppler compensation (see [5] for details). During the experiment a costal monitoring buoy measured wave heights of approximately 0.63 m with frequency between 0.43 and 0.40 Hz. Fig. 3(a) shows the ambiguity surface  $|P_{TR}(\Delta r, \Delta \varsigma_A; \Delta \omega)| = |P_{TR}(\Delta t; \Delta \omega)|$ . The dashed-line is the estimated compensation computed from GPS data using the waveguide invariant property as in [2], assuming that only a range shift is present. The continuous line represents the ambiguity surface maximum along time and frequency shift. Fig. 3(b) shows the power spectrum density of the ambiguity surface along the maximum trajectory, where one can see a peak at 0 Hz that is due to the continuous range shift compensation and a clear side lobe at 0.39 Hz that is in good agreement with the surface wave frequency. Such results allow one to speculate that the frequency shift is simultaneously compensating for the continuous range drift and for the depth swinging of the surface suspended array.

### **4. FUTURE WORK**

In the pTR context, real data and simulations corroborate the hypothesis that a frequency shift is able to compensate for the geometric mismatches in the vicinity of its nominal values, even at high frequencies. For the range mismatch such compensation is based on the known waveguide slope invariant. For the source and array depths (due to the similarities with the range case) it is expected that other two new waveguide invariants are involved. Future work will address the theoretic foundations of those new invariants and will explore their usefulness in the development of a physical-based equalizer for underwater communications.

## **5. ACKNOWLEDGEMENTS**

This work was supported by Fundação para a Ciência e a Tecnologia under NUACE POSI/CPS/47824/2002 project. The authors would like to thank the NATO Undersea Research Centre for the organization of MREA'04 sea trial, NRV Alliance master and personnel and the scientist in charge Emanuel F. Coelho.

#### REFERENCES

- [1] S.M. Jesus, C. Soares, A. Silva, J.P. Hermand, E. Coelho, AOB-Acoustic Oceanographic Buoy: concept and feasibility, In *UDT'06*, Hamburg Germany, volume, pp. pages, 2006.
- [2] H.C. Song, W.A. Kuperman, W.S. Hodgkiss, A time-reversal mirror with variable range focusing, JASA, 103 (6), pp. 3234-3240, 1998.
- [3] S.C. Walker, P. Roux, W.A. Kuperman, Focal depth shifting of a time reversal mirror in a range-idependent waveguide, *JASA*, 118 (3), pp. 1341-1347, 2005.
- [4] S. Jesus, C. Soares, P. Felisberto, A. Silva, L,Farinha, C. Martins, Acoustic maritime rapid environment assessment during the MREA04 sea trial, *Tec. Rep. 02/05 CINTAL-Universidade do Algarve*, 2005 (www.deei.fct.ualg.pt/SiPLAB/)
- [5] J. Gomes, A. Silva, S.M. Jesus, Joint passive time reversal and multichannel equalization for underwater communications, In Proc. of *MTS/IEEE OCEANS'06*, 2006.