

## Three-Dimensional Eigenray Search for a Vertical Line Array

Rogério Calazan<sup>a</sup>, Orlando Rodríguez<sup>b</sup>

<sup>a,b</sup>Signal Processing Laboratory (SiPLAB), LARSyS, University of Algarve, Campus de Gambelas, 8005-139 Faro, Portugal

Rogério Calazan, SiPLAB, LARSyS, University of Algarve, Campus de Gambelas, 8005-139 Faro, Portugal

fax number: (351) 289800949

email address: a53956@ualg.pt<sup>a</sup>, orodrig@ualg.pt<sup>b</sup>

**Abstract:** *Eigenray search in a two-dimensional waveguide is a feasible task, that can be solved efficiently through optimization of an ad hoc defined function (like, for instance, final ray depth deviation from hydrophone depth) over original ray elevations; in the worst case even when rays can be reflected backwards to the source an exhaustive search over the ray trajectory can be able to identify eigenrays by testing the proximity of each ray to the hydrophone. In three-dimensional waveguides the situation is far more complicated: the proximity method can be found to be very inefficient due to out-of-plane propagation and optimization needs to be developed over the plane of ray elevations and azimuths. The work presented here addresses the three-dimensional search of eigenrays based on the simplex method, implemented in a recent ray model; the performance of the method is discussed through comparisons with experimental data for a vertical line array.*

**Keywords:** *Three-dimensional eigenray search, Simplex optimization, Vertical line array*

## 1. INTRODUCTION

Eigenray search in two dimensions is a feasible task, which has been widely solved in ray tracing models in order to predict a channel impulse response. However, in real conditions, non linear internal waves and bathymetric features can lead to horizontal propagation effects which requires a three-dimensional model to produce a reliable prediction. Eigenray search in this context is a complicated problem due to the need to deal with ray trajectories in three-dimensions and the associated computational burden [1,2], even when considering a single hydrophone [3,4]. The problem becomes more demanding if one takes into account that real applications generally rely on the deployment of vertical line arrays to sample the propagating wave. This work proposes a three-dimensional search of eigenrays based on simplex optimization. It is shown that the proposed method predicts the channel impulse response in a feasible time; the performance of the method is also discussed through comparisons with experimental data for a vertical line array (VLA).

## 2. SIMPLEX BASED EIGENRAY SEARCH

Generally speaking, the simplex method allows to optimize a function with  $N$  variables using a geometric figure consisting of  $N + 1$  points or vertices [6]; in two dimensions the simplex is just a triangle. After the initial simplex has been computed an interactive procedure is started using operations named reflection, contraction and expansion, which are driven by the optimization itself. For the case of three-dimensional eigenray search the optimization is started by launching an initial set of rays, needed to sweep the waveguide. Such rays are used to perform a candidate selection, which is critical to reduce the computational time. The search starts sequentially selecting four rays with launching angles  $(\theta_i, \varphi_j)$ ,  $(\theta_i, \varphi_{j+1})$ ,  $(\theta_{i+1}, \varphi_j)$  and  $(\theta_{i+1}, \varphi_{j+1})$ , where  $\theta$  and  $\varphi$  represent angles of ray elevation and ray azimuth, respectively. Next, the elevation angles are kept fixed, and the azimuth index starts to be incremented until a hydrophone is located between the rays or when the last index is reached, followed by an increment of the elevation index and restart of the azimuth search. When a hydrophone is located between the rays the launching angles are selected as input for simplex based optimization. With such candidate space selected, an initial simplex is defined using three points; for each point the Eikonal equations are solved and the Euclidian distance between the ray and the hydrophone position are used as the function to be optimized. To find the smallest distance from the ray to the hydrophone a vertical plane is calculated, using the normal vector between the source and the hydrophone to intercept the corresponding ray coordinate. If the distance is less than a given threshold the procedure is finished and the launching angles are used to calculate the eigenray. A problem that arises in this context is how to bracket an optimal point when the search space isolation is not guaranteed. In fact, eigenray repetition can potentially lead to a false prediction and need to be avoided. To this end the algorithm stores information about previously found eigenrays (launching angles, number of surface and bottom reflections, etc.), that is used to compare with eigenrays found in the vicinity of the search space. If a previous eigenray with the same characteristics as the new one is found the new eigenray is discarded, and the next search is started. The above procedure of simplex based three-dimensional eigenray search was implemented in TRACEO3D, which is a three-dimensional extension of the TRACEO ray

model [5]; TRACEO3D is under current development at the Signal Processing Laboratory (SiPLAB) of the University of Algarve.

### 3. CALCOM'10 EXPERIMENTAL DATA

The CALCOM'10 sea trial took place in the period of 22th to 24th June 2010 at the south coast of Portugal, about 12nm south-east of Vilamoura [7]. Fig. 1(b) shows a bathymetry map whit the geometric setup of the experiment performed during day 3. The squares represent the point of deployment (A2d) and recovery (A2r) of the VLA, which drifted along the black curve. The dotted line represents the ship/source GPS track. Six events labelled as P1, P2, P3, P4, P5, P6 were conducted with several acoustic transmissions. For this work the model predictions were tested using experimental data from the event P3, with transmissions of a linear frequency modulated (LFM) signal in the band of 500-1000 Hz. The VLA considered in this analysis is an Acoustic Oceanographic Buoy (AOB), consisting of 16 hydrophones equally spaced at 4 m, with the first hydrophone approximately at 6.3 m from the sea surface, and the deepest about 66.3 m depth. The water temperature was acquired by the temperature sensors array of the VLA along time and, according to the transmission time, the temperature data was selected to compute a mean sound speed profile, which is shown in Fig. 1(a).

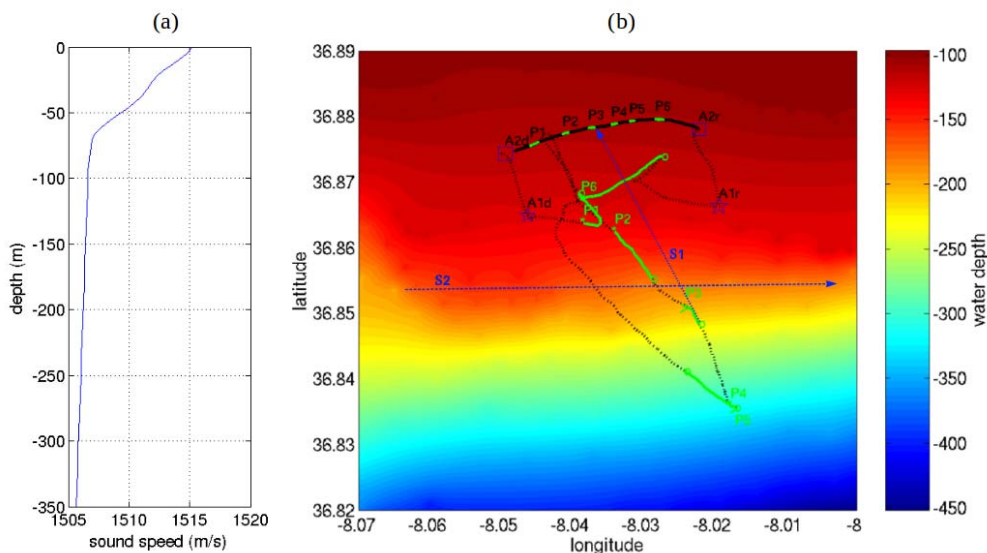


Fig. 1: (a) CALCOM'10 mean sound speed profile; (b) experimental site, with GPS estimated locations of AOB deployment and recovery (A2d,A2r), ship/source track during transmission events (green lines) and idealized track of transmissions (dashed blue line).

### 4. RESULTS AND ANALYSIS

Predictions with TRACEO along two-dimensional transects, and TRACEO3D using the full bathymetry and simplex based three-dimensional eigenray search were carried out in order to model the acoustic channel impulse response for the conditions of event P3, corresponding to the direction S1 shown in Fig. 1(b). The results are compared with the experimental data in order to investigate out-of-plane effects. Range corresponds to 3462 m, bottom depth at source position is 213 m and 111 m for the VLA position. The source range was derived from GPS information and the source depth from the pressure sensor at the source. Bottom depths for the source and VLA were calculated from bathymetry data. The

bottom parameters are provided in [7], indicating that sediment sound speed corresponds to 1650 m/s, bottom density is 1.7 g/cm<sup>3</sup> and attenuation is 1.0 dB/λ. The predicted two-dimensional and three-dimensional channel impulse responses for the VLA are shown in Fig. 2(a) and (b), respectively.

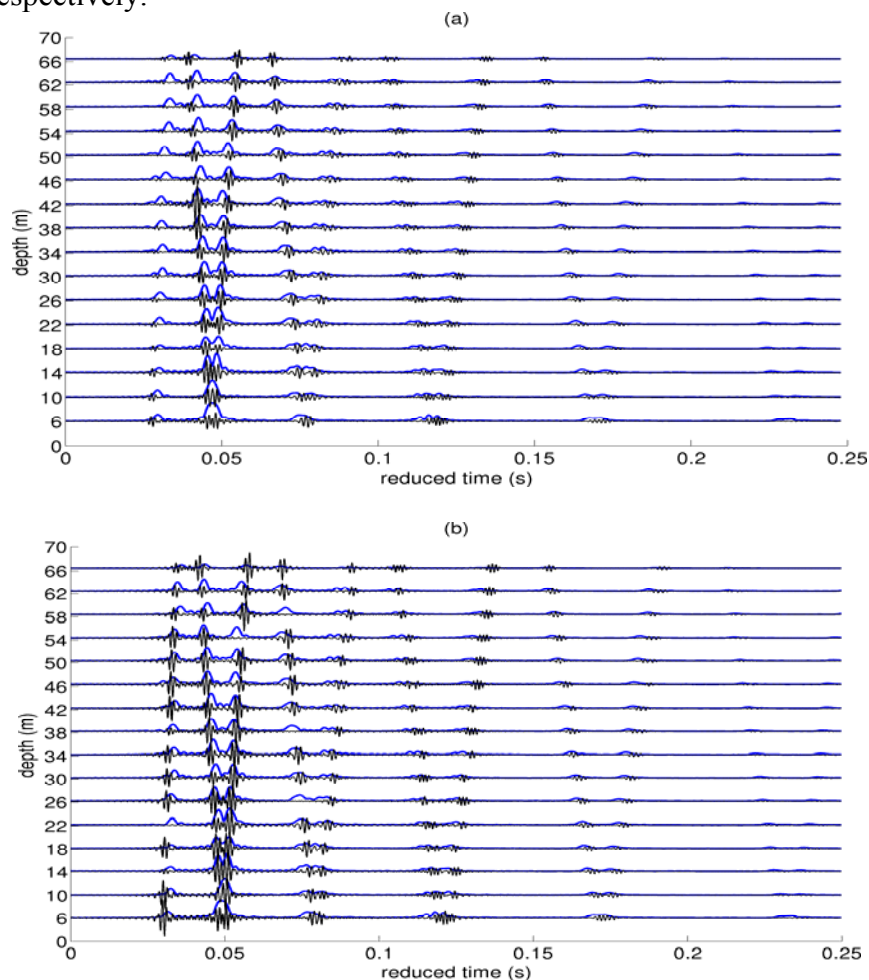


Fig. 2: Predicted channel impulse responses for the VLA (black lines) with: (a) TRACEO; (b) TRACEO3D (using simplex based eigenray search). Both predictions are plotted on top of experimental data (blue line).

The two predictions are plotted on top of the same experimental data. Generally speaking, both TRACEO and TRACEO3D seem able to produce a reliable prediction of the channel impulse response. There are, however, important differences. While the travel times are similar in both cases the first and second arrivals are better predicted in amplitude with TRACEO3D. Additionally, for later arrivals (mainly after the 5th) the travel times start to differ, and some channels predicted with TRACEO3D show a better agreement than those predicted with TRACEO; TRACEO predictions also appear slightly delayed compared to the experimental data. This can be explained by taking into account that 3D eigenrays can expend less time arriving at the hydrophones by following an upslope/downslope path and partially propagating over a bottom with a depth, smaller than the one travelled along a two-dimensional transect. This explanation is well supported by Fig. 3, which shows the arrival patterns for hydrophone 2, located at 10.3 m depth; the figure clearly indicates differences in travel times predicted with TRACEO and TRACEO3D; the corresponding eigenrays for the hydrophone 2 are shown in Fig. 4, and exhibit pronounced out-of-plane trajectories. Three-dimensional model predictions indicate that the maximum azimuth deviation is about 2°, relative to the source-hydrophone line of sight, which is achieved with  $\theta \approx 15^\circ$ . The fact that

the differences in travel times are so subtle is believed to be the result of upslope propagation in the P3 event, which despite the small range takes place along a significant gradient of bottom depth, masking the out-of-plane effects. This explanation is supported with additional modelling for the idealized track S2 of transmissions, shown in Fig.1(b), with the source and the VLA well aligned in a cross-slope direction. Source-hydrophone range corresponds to 5500 m, bottom depth at the source and VLA positions are 175 m and 181m, respectively, indicating that the gradient of bottom depth is negligible for such track. The maximum azimuth deviation obtained from the calculations corresponds to about  $5^\circ$ , for an elevation angle  $\theta \approx 19^\circ$ . TRACEO and TRACEO3D amplitude and delay predictions are shown in Fig. 5, and clearly indicate that after the second arrival eigenrays calculated with TRACEO3D take less time to propagate than those predicted with TRACEO, and the difference between the predictions increases for later arrivals.

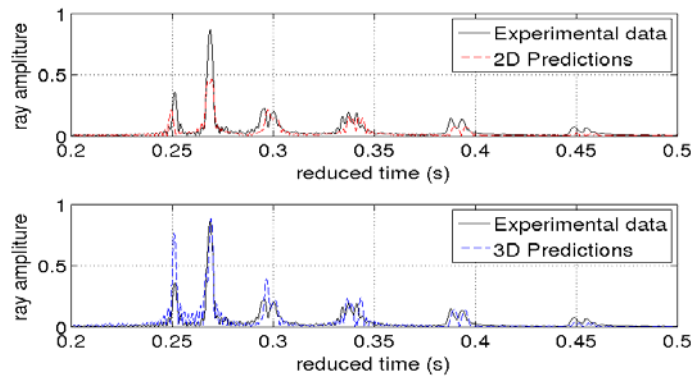


Fig. 3: Arrival patterns regarding hydrophone at 10.3 m depth for: (a) TRACEO; (b) TRACEO3D.

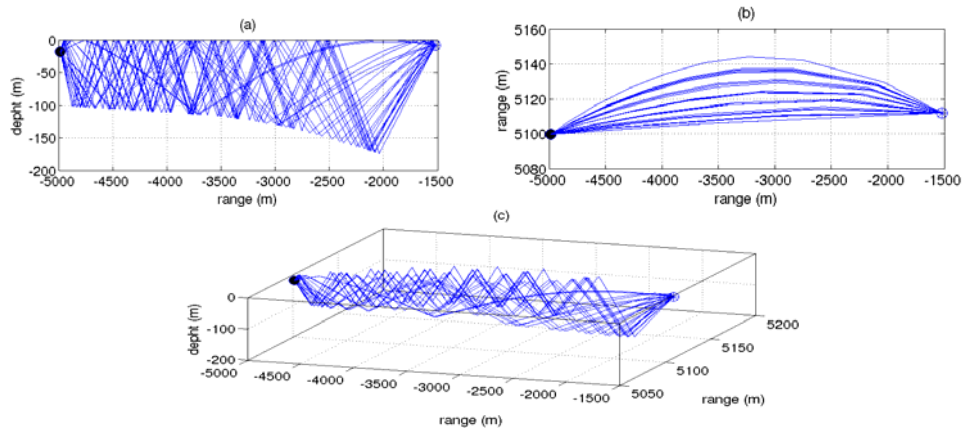


Fig. 4: Eigenray plots for hydrophone at 10.3 m depth from the surface for: (a) the vertical plane; (b) the horizontal plane; (c) perspective view. Notice that in (b) the x axis corresponds to 3.5km and the y axis corresponds to 80m.

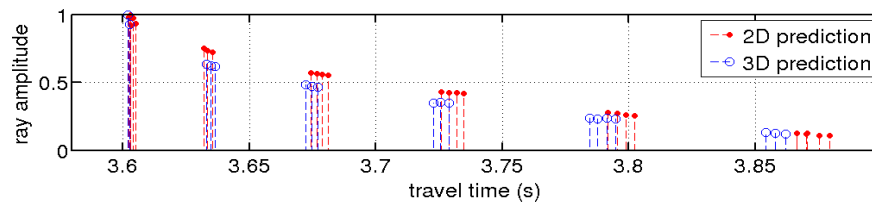


Fig. 5: Amplitudes and delays for 2D prediction in red and 3D prediction in blue regarding to a simulation using a hypothetical source-hydrophone position.

Another important issue is related to the efficiency of the simplex method: in the original version of TRACEO3D eigenrays were calculated by proximity; in other words, by launching a large number of rays one could expect to discover some of them, being close enough to the hydrophone. Application of the proximity method to produce predictions of the VLA experimental data failed completely, even when computing as much as one million rays. Such calculations took approximately 2776.9 s, while the (successful) simplex based method took only 85.3 s; TRACEO predictions took only 6.1 s, certainly less than TRACEO3D, but yet not necessarily the most accurate.

## 5. CONCLUSIONS

This paper presented a three-dimensional eigenray search based on simplex optimization for a VLA. When compared with the proximity method, the proposed search is much more efficient in terms of accuracy and runtime. Although significant out-of-plane effects can not be unquestionable verified in the analysis of experimental data there is important evidence of three-dimensional effects being relevant. Moreover, the experimental data provided an important basis to assess the performance and efficiency of the simplex based eigenray search method. Furthermore, simulation results using an idealized track of transmissions suggests that out-of-plane effects can be expected to be more intense in cross-slope direction with small bottom depth gradients, with upslope propagation being able to mask such effects.

## 6. ACKNOWLEDGEMENTS

This work received support from the Foreign Courses Program of CNPq and the Brazilian Navy. The authors would like to express their thanks to Prof. Paulo Felisberto for providing assistance in understanding the data from the experiment.

## REFERENCES

- [1] **Jensen, F.B., Kuperman, W.A., Porter, M.B., Schmidt, H.**, *Computational Ocean Acoustics*, Springer, 2<sup>o</sup> ed, 2011.
- [2] **Sturm et al.**, Numerical investigation of out-of-plane sound propagation in a shallow water experiment, *J. Acoust. Soc. Am.*, 124(6), 2647, 2008.
- [3] **Reilly, S. M., Potty, G. R., Goodrich, M.**, Computing Acoustic Transmission Loss Using 3D Gaussian Ray Bundles in Geodetic Coordinates. *Journal of Computational Acoustics*, 24(01), 2016.
- [4] **Xing, Chuan-xi, et al.**, Parallel computing method of seeking 3D eigenrays with an irregular seabed, *Acoustics in Underwater Geosciences Symposium (RIO Acoustics)*, IEEE/OES., IEEE, pp. 1-5 , 2013.
- [5] **Rodríguez et al.**, Seismo-acoustic ray model benchmarking against experimental tank data. *J. Acoust. Soc. Am.*, 132(2), 709–717, 2012.
- [6] **Nelder, J. A., Mead, R.**, A simplex method for function minimization, *The computer journal*, 7(4), 308-313, 1965.
- [7] **P. Felisberto, S.M. Jesus and F. Zabel**, CALCOM'10 Sea Trial, *Field data calibration report*, SiPLAB report, 2010.