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INTERNAL TIDE ACOUSTIC TOMOGRAPHY: RELIABILITY OF THE NORMAL MODES EXPANSION AS A POSSIBLE BASIS FOR SOLVING THE INVERSE PROBLEM

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Abstract: *Using hydrodynamic and thermodynamic equations appropriate for modelling internal tides, one can predict the current and temperature distributions associated with the ocean's dynamic modes. Comparing such predictions with observations from the INTIMATE'96 experiment, we find a high degree of correlation between the first 3 theoretically calculated dynamic modes and corresponding empirical orthogonal functions (EOF's) derived from an ensemble of temperature and current profiles. The implications are twofold. First, this implies that the dominant variability in the INTIMATE'96 experiment is indeed associated with internal tides. Secondly, it suggests that in future tidal experiments a theoretically generated basis may be used as effectively as an EOF basis (which requires more extensive oceanographic measurements). We have also used the set of dynamic modes to simulate the effect of the tides on acoustic propagation to understand the relative importance of the usual surface tide (barotropic) and the internal (baroclinic) tides.*

1 INTRODUCTION

Possible applications of OAT for inversion of the internal tide field have not been considered till the present time, despite the large amount of references related to the influence of internal tides on acoustic signals and simulations of underwater propagation through an internal wave field (see [1] for a particular analysis and a list of publications). Most of the studies consider as starting assumptions that: 1)the statistical distribution of the internal modes follows the GarrettMunk spectrum, and 2)the perturbations on sound velocity (and hence on temperature) can be represented as an orthogonal decomposition on the basis of dynamic normal modes (hereafter simply normal modes) of the internal wave. Both approximations simplify significantly the solution of the inverse problem and seem to offer a good description of acoustical propagation in deep water scenarios. However, the validity of those assumptions becomes dubious when applied to sound propagation in the continental shelf. A key complication is the affect of the tides which leads to an internal wave spectrum with strong peaks at the tidal frequencies. In addition, the internal waves are often of large amplitude leading to nonlinear effects and associated solitons or bores.

To understand these issues more completely an experiment called INTIMATE'96 (INternal Tide Investigation by Means of Acoustic Tomography Experiment) was conducted off the coast of Portugal. Oceanographic measurements conducted during the experiment have enabled us to perform a detailed analysis of hydrodynamic and thermodynamical equations governing the internal tides. As will be shown in the following sections the resulting current and temperature modes provide a detailed description of the internal tide and offer a reliable set of normal modes for oceanographic inversion.

2 THEORETICAL BACKGROUND

Internal tides are a particular case of internal waves of tidal frequency and generated by the interaction of the surface (barotropic) tide with bottom topography. The coupling mechanism remains poorly understood; however, the internal tides are generally most strongly excited at a sharp bathymetric feature such as as a sea mount or at the continental shelf-break. The main consequence of this interaction is the oscillation of the thermocline with the tidal period. However, the internal tides also have a surface manifestation. They cause very small displacements of the ocean level that can be measured by satellite altimeters. In addition they circulate organic surfactants disturbing the surface reflectivity at both optical and radio bands and revealing their banded structure.

The hydrodynamic equations for baroclinic currents [2], in the case of internal planewave propagation, can be solved by an eigenfunction expansion using a basis of functions, Ψj, and their derivatives, $\phi_j = d\Psi j/dz$, in the following way:

$$(u,v) = \sum_j (u_j, v_j)\phi_j \exp\left[i\left(k_x^j x + k_y^j y - \omega t\right)\right] \tag{1}$$

$$w = \sum_j wj\Psi_j \exp\left[i\left(k_x^j x + k_y^j y - \omega t\right)\right] \tag{2}$$

where ω corresponds to the tidal frequency. The eigenfunctions Ψ_j can be obtained by solving the standard differential equation for internal waves (see [2],p.223). Similar expansions can be used for pressure and density perturbations. From a 'geometrical' point of view the given set of expansions can be considered as projections of currents onto two different bases of eigenfunctions: w is projected onto the orthogonal basis formed by functions Ψ_j and (u,v) onto the corresponding basis of functions ϕ_j.

The wavenumber components are related to the direction of internal tide propagation θ through the relationships:

$$k_x^j = k_h^j \cos\theta \ , \ k_y^j = k_h^j \sin\theta \ ,$$

where k_h^j is the eigenvalue of the jth normal mode ($k_h^j < k_h^{j+1}$) and its inverse is proportional to the modal wavelength. Vertical stratification of the environment is represented in the differential equation for Ψ_j through the buoyancy frequency $N^2(z)$, which is normally related to mean density. For inversion it is better to use the alternative relationship

[2]

$$N^2 = g\left[a_T\frac{dT_0}{dz} + a_T^2\frac{gT_0}{C_{ps}} - a_s\frac{ds}{dz}\right] \ , \tag{3}$$

where $a_T = 2.4110^{-4}\,(^\circ\mathrm{C})^{-1}$ and $C_{ps} = 3994\ \mathrm{J(kg^\circ C)}^{-1}$. Usually the salinity depends weakly on depth so we can neglect the salinity term and develop a buoyancy profile that depends only on the temperature. Thus, the temperature profile provides a critical piece of information. It allows us to calculate the buoyancy profile and thereby the dynamical modes. The dynamical modes themselves are characterized in terms of their spatial and temporal scales and provide a suitable basis for expressing the ocean structure in terms of density, currents, and pressure. Nevertheless, from a tomographic point of view, the system of Eqs.(1)-(2) does not provide a dynamical basis for expanding the sound speed field. To address this, recall the thermodynamical equation [2]:

$$\frac{D}{Dt}\left(\rho c_v T\right) = \nabla\left(k_T \nabla T\right) + Q_T \tag{4}$$

where c_v denotes the specific heat at constant volume, k_T the thermal conductivity and Q_T represents all sources and sinks of heat. An approximate solution for temperature perturbations,

$$T - T_0(z) = \frac{dT_0}{dz}\sum_j T_j \Psi_j \exp\left[i\left(k_x^j x + k_y^j y - \omega t\right)\right] \tag{5}$$

can be obtained in the case of k_T, $Q_T = 0$ and assuming ρ and c_v are constant. This solution provides a physical relationship between the temperature field and the basis of normal modes; the orthogonal expansion for temperature (and hence for sound velocity) follows automatically from Eq.(5) when the temperature gradient depends weakly on depth.

3 COMPARISON WITH EXPERIMENTAL DATA

During the INTIMATE'96 experiment [3] an intensive survey of thermistor, CTD, XBT and ADCP data was conducted at the Vertical Array (VLA), and along two transmission legs, one due north and one due west of the array (see Fig.1(a)). Received signals were later correlated with an estimate of the transmitted waveform and then aligned and averaged over 10 transmissions (about 1 minute) to increase the signaltonoise ratio. The experiment was conducted near the shelf break were the internal tides tend to be strongest.

Both sets of normal modes and their derivatives (Fig.1(b)) were obtained from the mean profile of temperature at the VLA. The correlation between theoretical and empirical functions is shown in Fig.2(a) and Fig.2(b) for currents and temperature respectively. The correlation coefficients were estimated by expanding current and temperature EOF's onto corresponding bases of theoretical functions as follows:

$$EOF_l^{(u,v)} \approx \sum a_{lm}\phi_m \ , \ EOF_l^T \approx \frac{dT_0}{dz}\sum b_{lm}\Psi_m$$

where (u, v) is the horizontal current and T is the temperature. Since the basis is orthogonal, the coefficients in the above expansions are easily calculated by inner products. The results presented in Fig.2 reveal a strong correlation between empirical and corresponding theoretical eigenfunctions up to mode 3. This result is very significant since it indicates that every theoretical function from the given set is equivalent to the correspondent EOF. Furthermore, the quantity and resolution of empirical functions depends on the number and resolution of measured profiles, while all the theoretical functions can be obtained from a coarse estimate of the mean temperature distribution and still provide a detailed description of the internal tide.

In a simple model, the internal tides propagate like plane waves. Thus, we can also use the dynamical modes to estimate the direction of propagation of the internal tide by looking at how the modal amplitudes at two different locations are shifted with respect to each other. (A similar approach was applied in Ref.[4] using isotherms in the INTIMATE'96 experiment.) Calculated amplitudes of modal oscillations for u and v are shown in Fig.3(a). The time shift for every pair of modal oscillations was estimated by looking at maximizing the peak in crosscorrelations between coefficients at two different stations. The lags associated witht the first three modes were 2, 3, and 3 hours respectively giving a mean lag of 2.7 hours. Figure 3(b) allows us to convert this time lag to an angle of propagation yielding $\theta \approx 15°$. This is in close agreement with a theoretical prediction based on the orientation of the shelf break. These results are further supported in studies of the temperature coefficients at different stations.

4 ACOUSTIC SIMULATIONS AND REAL TRANSMISSIONS

The acoustic data taken in INTIMATE'96 shows a clear tidal cycle; however, since both the surface tide and the internal tides have the same temporal frequency it is not readily obvious which component is driving the acoustic perturbations. To study this further, we use the dynamical modes together with surface tide predictions, to calculate temperature and sound velocity distributions. We then simulate the impact of both barotropic and baroclinic tides on acoustic transmissions.

These simulations were performed with the KRAKEN model [5] for the lower hydrophone of the VLA located at 115 m depth, and suggest that the surface tide introduces periodic oscillations of the later groups of arrivals (Fig.4(a)) but not the early arrivals (Fig.4(c)). This situation changes when the internal tide is included in theoretical fields of temperature and sound speed as shown in Fig.4(d). Received transmissions from the INTIMATE'96 experiment show such oscillations of late arrivals (Fig.4(b)) and confirm the simulated effects of the surface tide on received signals [3]. Since the received transmissions are aligned by their leading edge, the analysis of initial arrivals did not support (or contradict) the prediction related to the internal tide. Nevertheless the given set of simulations provided a clear distinction of barotropic and internal tide perturbations on received signals.

5 CONCLUSIONS

On the basis of this analysis the following conclusions can be drawn: 1)the buoyancy profile (which characterizes the waveguide stratification) can be properly obtained from mean temperature distribution; 2)theoretical normal modes can be accurately calculated from above mentioned buoyancy profile; 3)both sets of normal modes and normal mode

(a) (b)

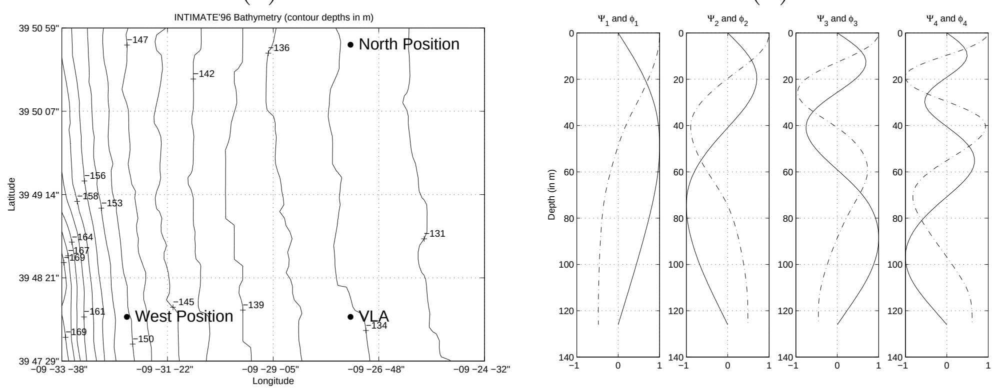

Figure 1: *(a)* Bathymetry of the INTIMATE'96 experiment; *(b)* temperature-derived normal modes (continuous line) and their derivatives (dott-dashed line).

(a) (b)

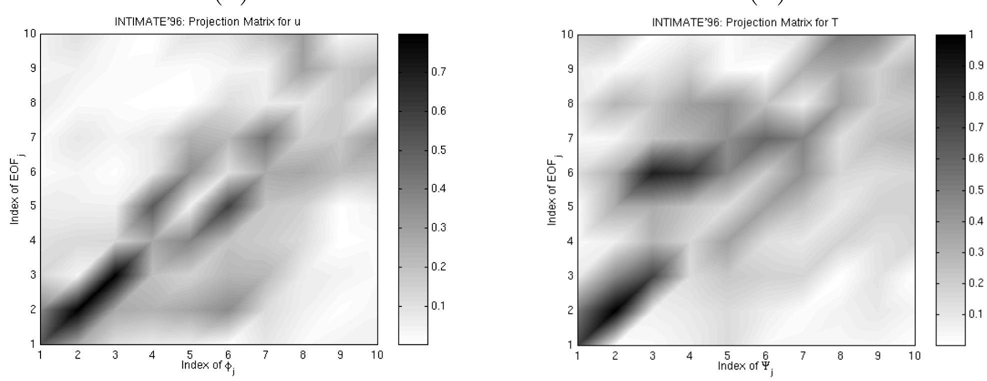

Figure 2: Correlation matrices for u *(a)* and temperature *(b)*.

(a) (b)

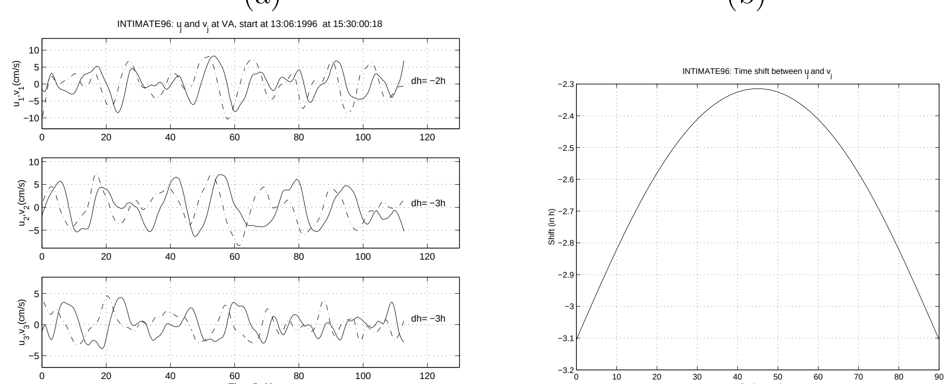

Figure 3: *(a)* Modal amplitudes for u (continuous line) and v (dott-dashed line); *(b)* theoretical dependence of phase shift on θ (note the time shift at $\theta = 15°$).

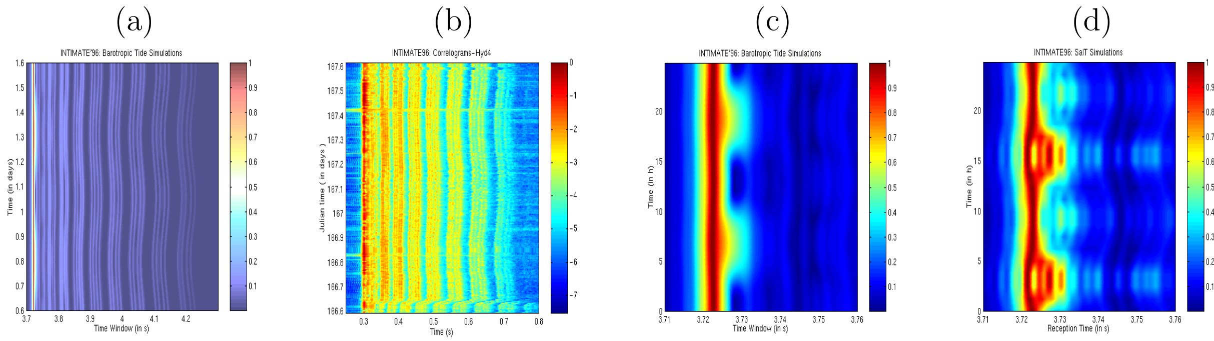

Figure 4: *(a)* Simulated arrival patterns for the surface tide and *(b)* real transmissions from the INTIMATE'96 experiment; zoom on initial peaks for surface tide *(c)* and both surface and internal tides *(d)*.

derivatives offer a detailed description of the internal tide, in particular through expansions of temperature and currents on corresponding basis of normal modes and normal mode derivatives, respectively; 4)the set of theoretical normal modes (and normal mode derivatives) is highly correlated with EOF's of temperature and current data, up to mode 3; 5)this set of normal modes can be used to generate physically consistent fields of temperature and sound velocity; 6)acoustic simulations based on such fields allow one to clearly distinguish the effects of both barotropic and internal tides on underwater acoustic transmissions by revealing oscillations of late and early arrivals; such oscillations can be seen for the case of the surface tide in real data from the INTIMATE'96 experiment.

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