A SENSITIVITY STUDY FOR FULL-FIELD INVERSION OF GEO-ACOUSTIC DATA WITH A TOWED ARRAY IN SHALLOW WATER

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ABSTRACT Inversion of acoustic data for estimating bottom acoustic parameters has been the subject of a considerable number of studies. Usually, signals are received on vertical arrays of sensors and transmitted from sound sources being towed away from the array location in order to form a syntethic aperture array. That configuration is greatly dependent on the knowledge of the source-receiver distance which is, in practice, relatively difficult to measure with the required precision. Also, since the vertical array is generally moored, or slowly drifting, the area that can be surveyed with such a method is limited to a tenth of a mile in shallow water. Changing area requires the recovery and redeployment of the whole system. This paper explores the possibility of using an horizontal array and a sound source simultaneously towed by a single ship where the source-receiver distance is constant. It has been shown that sensitivity to sound speed variations is higher on the first bottom layers and it increases with array length. Density and attenuation (both compressional and shear) have in general little influence on the acoustic field structure and are therefore difficult to estimate. Increasing the signal frequency bandwidth by incoherent module averaging has no significant influence on sensitivity. Mismatch cases, mainly those related to array/source relative position, showed that deviations of more than $\lambda/2$ in range and $\lambda/5$ in depth may give erroneous *extremum* location and therefore biased final estimates.

1. Introduction

Estimating ocean bottom morphological characteristics in coastal shallow waters is of considerable economical and scientific interest. Sound propagation in shallow water is known to be dominated by a strong signal interaction with the medium boundaries. In principle, comparison of the received acoustic field with that predicted by a suitable propagation model would allow for estimating bottom and surface physical properties. Estimation of bottom environmental parameters from the acoustic data received on an array of sensors is known to be an ill-conditioned inversion problem for which an analytical solution is unknown. Ill-conditioning strongly depends on the non-linearity of the function to be inverted and on the dimension of the parameter space. Brute force inversion, by extensive forward modeling exploration of the whole search space, has been widely used on matched-field processing for source localization, after the pioneering work of Bucker [1]. Using this approach for geo-acoustic data inversion would be computationally prohibitive due to the high dimension of the parameter space to be searched. Therefore, the alternative taken by several authors, combines a matched-field type of technique (its output is a multidimensional ambiguity surface), with a powerful search algorithm that, in principle, allows a quick convergence to the *extremum* of that surface [2] [3] [4]. Simply speaking, the matched field technique provides a parameter dependent cost function that the search algorithm attempts to minimize. In

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other words, the treatment of the inverse problem has two aspects: one is the computing of the cost function and the other is the search algorithm.

The present study concentrates on the cost function system dependence and on its operational characterization. Previous studies used either vertical arrays or sound sources being towed away from the receiver location in order to create a synthetic aperture and resolve mode arrivals. In principle, a shipborne only system would allow easier deployment and lower cost for surveillance of large areas. In our study it is assumed that the ship is towing both the source and the array, such that the source-receiver range is constant. To obtain an idea of the expected performance of the system and draw some conclusions on its operation, this study presents the cost function sensitivity to variations of: array length (spacing and number of sensors), source depth, receiver depth, source range, sensor noise, source frequency and frequency band. The canonical case consists of a 64 hydrophone - 4 m spacing towed array at 100 m depth and a harmonic 100 Hz source also at 100 m depth and at 200 m range.

2. Theoretical background

2.1. THE DATA MODEL

The deterministic sound pressure at the receiver location r_l, z_l is modeled as the solution of the wave equation for a narrowband point source exciting a horizontally stratified range-independent environment:

$$p_l(\omega_k, r_l, z_l, z_0; \gamma) = \int_0^\infty g(\kappa, \omega_k; \theta_l, \gamma, z_0) J_0(\kappa r_l) \kappa d\kappa, \qquad (1)$$

where l denotes the lth array sensor, ω_k is the kth frequency bin, z_0 is the source depth and γ is a vector containing all the pertinent environmental parameters under estimation. Thus, at time snapshot n, the L sensor array received acoustic pressure, can be modeled as a multivariate complex normally distributed random variable

$$\mathbf{y}_n(\omega_k, \gamma_T) = b_n(\omega_k)\mathbf{p}(\omega_k, \gamma_T) + \epsilon_n(\omega_k) \qquad k = 1, \ldots, K \qquad (2)$$

where ϵ is the sensor noise assumed to be zero mean and uncorrelated both in time and from sensor to sensor. The scalar b_n is a complex random variable that accounts for the non-deterministic amplitude variation at the receiver due to the environmental inhomogenities and fluctuations that are not included in the sensor noise. Subscript $_T$ denotes the true environmental parameter value under estimation.

2.2. THE BROADBAND CONVENTIONAL MATCHED-FILTER

Given model (1)-(2), the problem is to detect, at each single frequency, a known signal $\mathbf{p}(\omega_k, \gamma)$ in white noise, for which the optimal processor is the matched-filter. Let

$$\Phi_{\mathrm{CMF}}(\omega_k, \gamma) = |\mathbf{y}(\omega_k, \gamma_T)^H \mathbf{p}(\omega_k, \gamma)|^2 \qquad \gamma \in \Gamma \qquad (3)$$

be the matched-filter output based on model replica prediction for search parameter γ with Γ denoting the whole environmental parameter search space. Thus, the final broadband optimal detector of a single known signal in white noise will be

$$\hat{\gamma}_T = \arg\max_{\gamma} \frac{1}{K} \sum_{k=1}^{K} |\sigma_s^2(\omega_k)|^2 \Phi_{\mathrm{CMF}}(\omega_k, \gamma) \qquad (4)$$

where $\sigma_s^2(\omega_k)$ is the source power at frequency ω_k.

2.3. CORRELATION OF DIRECTIONAL DATA

A common problem encountered when analysing geo-acoustic data is the superposition of the direct path source arrival with the bottom reflected data of interest. A possibility to separate those arrivals is by analysing the data in the wavenumber space domain and by filter out the direct path arrivals. For an horizontal array the arrivals associated with the steepest vertical angles, which are those that have a stronger interaction with the bottom, correspond to those arriving closer to broadside. That approach implies a transformation of the acoustic data from the hydrophone space to the wavenumber space which may be expressed by the Green's function. In practice, since the acoustic pressure is a discrete function defined over a finite array aperture, it implies that an estimate of the predicted Green's function can be given by

$$\hat{g}_p(k_j, \omega, \gamma) = \frac{e^{i\pi/4}}{\sqrt{2\pi k_j}} \sum_{l=1}^{L} p_l(\omega, \gamma) e^{-ik_j r_l} \sqrt{r_l}, \qquad j = 1, \ldots, N_w \tag{5}$$

where the discretization over the wavenumber space has been arbitrarily performed over N_w equally spaced points in $[0, 2\pi/d]$, d being the array sensor spacing (assumed constant). A one-to-one mapping from the wavenumber to the bearing space may be performed using $k_j = (2\pi f/c)\cos(\theta_j + \pi/2)$ for $\theta_j \in [-90°, +90°]$. With that definition -90° direction is aft (towards the source) and +90° is end fire. A similar expression to (5) may be used for the received data Green's function at time-snapshot n giving $\hat{g}_{y,n}(k_j, \omega, \gamma_T)$. Obviously, when computing (5) there is a windowing spatial effect that actually reduces the array resolution power. Based on (5) and once a given bearing sector $\theta_j \in [\theta_l, \theta_h]$ has been selected, a possible cost function can be defined as

$$\Phi_{\text{WS-CMF}}(\omega_k, \gamma) = |\sum_{j=j_l}^{j_h} \hat{g}_y^*(\theta_j, \omega_k, \gamma_T)\hat{g}_p(\theta_j, \omega_k, \gamma)|^2 \tag{6}$$

and an incoherent broadband function can be defined as the average of (6) over the required frequency band as

$$\Phi_{\text{WS-CMF}}(\gamma) = \frac{1}{k_h - k_l + 1} \sum_{k=k_l}^{k_h} \Phi_{\text{WS-CMF}}(\omega_k, \gamma). \tag{7}$$

Expression (6) has been written as time independent for a question of generality, where in practice it is time-snapshot dependent. In practice time averaging is often performed over a "reasonable" time window where the acoustic field is assumed to be stationary.

2.4. BROADBAND MAXIMUM-LIKELIHOOD MATCHED-FIELD

Given data model (1)-(2) the random complex scalar $b_n(\omega_k)$ is assumed random $N(0, \sigma_b^2)$. The sensor noise $\boldsymbol{\epsilon}_n(\omega_k)$ is assumed also $N(\mathbf{0}, \sigma_\epsilon^2\mathbf{I})$. With these assumptions, at time snapshot n, the observation vector is therefore $N(b_n(\omega_k)\mathbf{p}(\omega_k, \gamma_T), \sigma_\epsilon^2\mathbf{I})$. At this point one desires to determine the "best estimator" of γ_T given the set of broadband vectors $\mathbf{Y}^n =$

$[\mathbf{y}_n^T(\omega_1), \mathbf{y}_n^T(\omega_2), \ldots, \mathbf{y}_n^T(\omega_K)]^T$. If the time interval T, used for calculating each individual Fourier transform, is such that $T >> \tau_0$, where τ_0 is the correlation time of the most coherent signal or noise, the vector $\mathbf{Y}^n$ has a near-block-diagonal covariance matrix because the Fourier coefficients at different frequencies are asymptotically uncorrelated. Independently from the nature of the assumed data model the log-likelihood function is given by

$$L(Y) = -\sum_{n=1}^{N}[(\mathbf{Y}^n - \boldsymbol{\mu}^n)^H \mathbf{R}_n^{-1}(\mathbf{Y}^n - \boldsymbol{\mu}^n) + \log\det(\pi\mathbf{R}_n)] \tag{8}$$

where ^{H} denotes Hilbert transpose, $\boldsymbol{\mu}^n = E[\mathbf{Y}^n]$ and $\mathbf{R}_n = \mathrm{COV}[\mathbf{Y}^n]$ which is, with the assumptions above a diagonal matrix. Inserting those two quantities into (8) one easily obtains

$$L(Y) = -\sum_{n=1}^{N}\sum_{k=1}^{K}\{\frac{1}{\sigma_\epsilon^2(\omega_k)} \parallel \mathbf{y}_n(\omega_k) - \mathbf{b}_n(\omega_k)\mathbf{p}(\omega_k, \gamma_T) \parallel^2 - L\log[\pi\sigma_\epsilon^2(\omega_k)]\} \tag{9}$$

Since $\mathbf{p}(\omega_k, \gamma_T)$ is given by (1), for the search parameter γ as $\mathbf{p}(\omega_k, \gamma)$, the only unknown is the random component $\mathbf{b}_n(\omega_k)$. An usual assumption is to introduce a least-squares estimate of the signal, which in that case is given simply by the projection of $\mathbf{y}_n(\omega_k)$ onto the vector $\mathbf{p}(\omega_k, \gamma)$ given by

$$\hat{\mathbf{b}}_n(\omega_k) = \frac{\mathbf{p}(\omega_k, \gamma)^H \mathbf{y}_n(\omega_k)}{\parallel \mathbf{p}(\omega_k, \gamma) \parallel^2} \tag{10}$$

Thus, introducing (10) into (9) one gets the estimator expression

$$\hat{\gamma}_T = \arg\ \min_{\gamma} \sum_{n=1}^{N}\sum_{k=1}^{K}\{\frac{1}{\sigma_\epsilon^2(\omega_k)} \parallel \mathbf{y}_n(\omega_k) - \frac{\mathbf{p}(\omega_k, \gamma)^H \mathbf{y}_n(\omega_k)}{\parallel \mathbf{p}(\omega_k, \gamma) \parallel^2}\mathbf{p}(\omega_k, \gamma) \parallel^2\} \tag{11}$$

and after some straight forward manipulat.ons and noting that the only n snapshot dependent quantities are the observation vectors $\mathbf{y}_n(\omega_k)$ one can rewrite (11) as

$$\hat{\gamma}_T = \arg\ \min_{\gamma} \sum_{k=1}^{K} \frac{K}{\sigma_\epsilon^2(\omega_k)}\ \mathrm{tr}\{[\mathbf{I} - \frac{\mathbf{p}(\omega_k, \gamma)\mathbf{p}(\omega_k, \gamma)^H}{\parallel \mathbf{p}(\omega_k, \gamma) \parallel^2}]\hat{\mathbf{R}}(\omega_k)\} \tag{12}$$

where the matrix $\hat{\mathbf{R}}(\omega_k)$ is the data sample covariance matrix estimate at frequency ω_k given by the time snapshot average of the data outer products. The only unknown quantity is the noise power over the required frequency band, $\sigma_\epsilon^2(\omega_k); k = 1, \ldots, K$ that can be assumed constant for white noise.

3. Simulation results

The simulation environment is shown on table 1. A computer code based on SAFARI, FIPP [5] has been developed to implement the cost functions defined above and necessary looping for all the environmental search parameters. The system scenario includes a 64 hydrophone array at 4 m spacing with a 100 Hz sound source both at 100 m depth and at 200 m range. Testing was performed with the conventional matched-filter approach and in one case a comparison was made with the other cost functions.

TABLE 1. Canonical environment

Depth(m)	P vel.(m/s)	S vel.(m/s)	P att.(dB/λ)	S att.(dB/λ)	Dens. (g/cm^3)
0.0	1500	0.0	0.0	0.0	1
140	1550	130	0.1	1.7	1.49
145	1700	350	0.8	2.0	1.88
150	2500	900	0.01	0.01	2.4

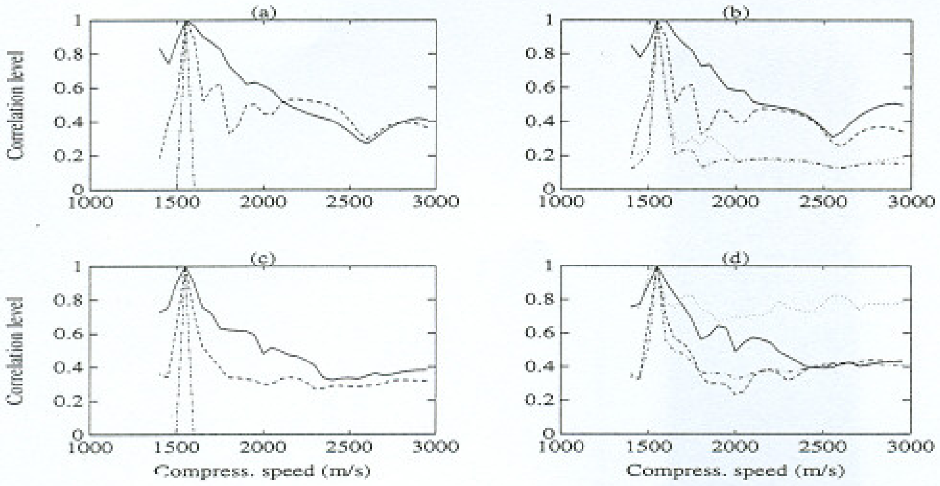

Figure 1: Correlation level versus compressional speed of first sediment layer for CMF (- - dashed), WS-CMF (– solid), LMS (... dotted) and ML (-.-. dash-dot). Narrowband 100 Hz in (a) without noise and (b) SNR = 10 dB; broadband 85 to 115 Hz in (c) without noise and in (d) with SNR = 10 dB.

Array aperture: the results show that varying array aperture from 63 m up to 2016 m increases the sensitivity to bottom parameters, mainly to the shallower sediment layers. Attenuation sensitivity is of the order of 10^{-3}. *Frequency*: varying source frequency between 25 and 200 Hz does change the angle of incidence and signal penetration into the bottom. An increase of frequency improves the sensitivity to compressional parameter variation while for deeper layers lower frequencies give better results. Shear parameters showed a higher sensitivity at lower frequencies that provided also smoother curves (less *minima/maxima*). *Bandwidth*: varying the source bandwidth between 2 and 60 Hz showed that there is no increase in sensitivity with, however, a higher smoothness of the cost function behaviour. *Source-receiver positions*: changing relative source-receiver depth does change the sensitivity curve according to the higher or lower transfer of energy between source and receiver that is depending on the mode excitation *vs* depth. Changing source receiver range has a similar behaviour depending on the modal interference pattern *vs* range. In both cases placing the source and the array at high energy transfer locations does improve the sensitivity. *System parameters mismatch*: higher sensitivity to depth than to range mismatch was observed. The accuracy to which sensor depth should be known has to be better than a $\lambda/5$ while an accuracy of $\lambda/2$ will be enough for sensor range. *Signal-to-noise ratio*: the narrowband and broadband performance of the algorithms was compared and the results are shown in figure 1. For reference the well known least mean squares (LMS) cost function has also been calculated and is shown in figure 1. Since the working signal to noise ratio (SNR) will be relatively high, only the SNR=∞, in (a) and (c), and 10 dB, in (b) and (d), are shown. In both cases the maximum likelihood (ML) estimate provided the best results together with the MLS estimator for SNR=∞ and with the conventional matched-filter (CMF) for SNR=10 dB. At lower SNR (not shown) CMF provided the highest performance. There was only a slight increase in performance with increasing bandwidth.

4. Conclusion

It was demontrated that cost function sensitivity to sound speed variations is higher to the bottom layers and it increases with array length. An increased sensitivity is generally accompanied by a cost function with non-monotonic behavior creating local *minima* and making it problematic to reach the global minimum. Density and attenuation (both compressional and shear) have in general little influence on the acoustic field structure and are therefore difficult to estimate. Increasing the signal frequency bandwidth by incoherent module averaging has no significant influence on sensitivity. A cost function relaying on the conventional matched filter has shown low sensitivity to sensor noise and has been extended to match directional data from bottom arrivals at several frequencies. A technique for providing a maximum likelihood broadband estimate of the peak location has been derived and showed a discrete performance at high SNR. Mismatch cases, mainly those related to array/source relative position, showed that deviations of more than $\lambda/2$ may give erroneous *extremum* location and therefore biased final estimates.

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