Workshop on Inverse Obstacle Problems

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by

Sérgio M.M. Jesus
SiPLAB - FCT, Universidade do Algarve
Campus de Gambelas, PT-8000-810 Faro, Portugal
sjesus@ualg.pt

Model-based inverse problems in underwater acoustics

S.M. Jesus
(sjesus@ualg.pt)

SiPLAB-FCT, Universidade do Algarve
Campus de Gambelas, PT-8000-810 Faro
Portugal

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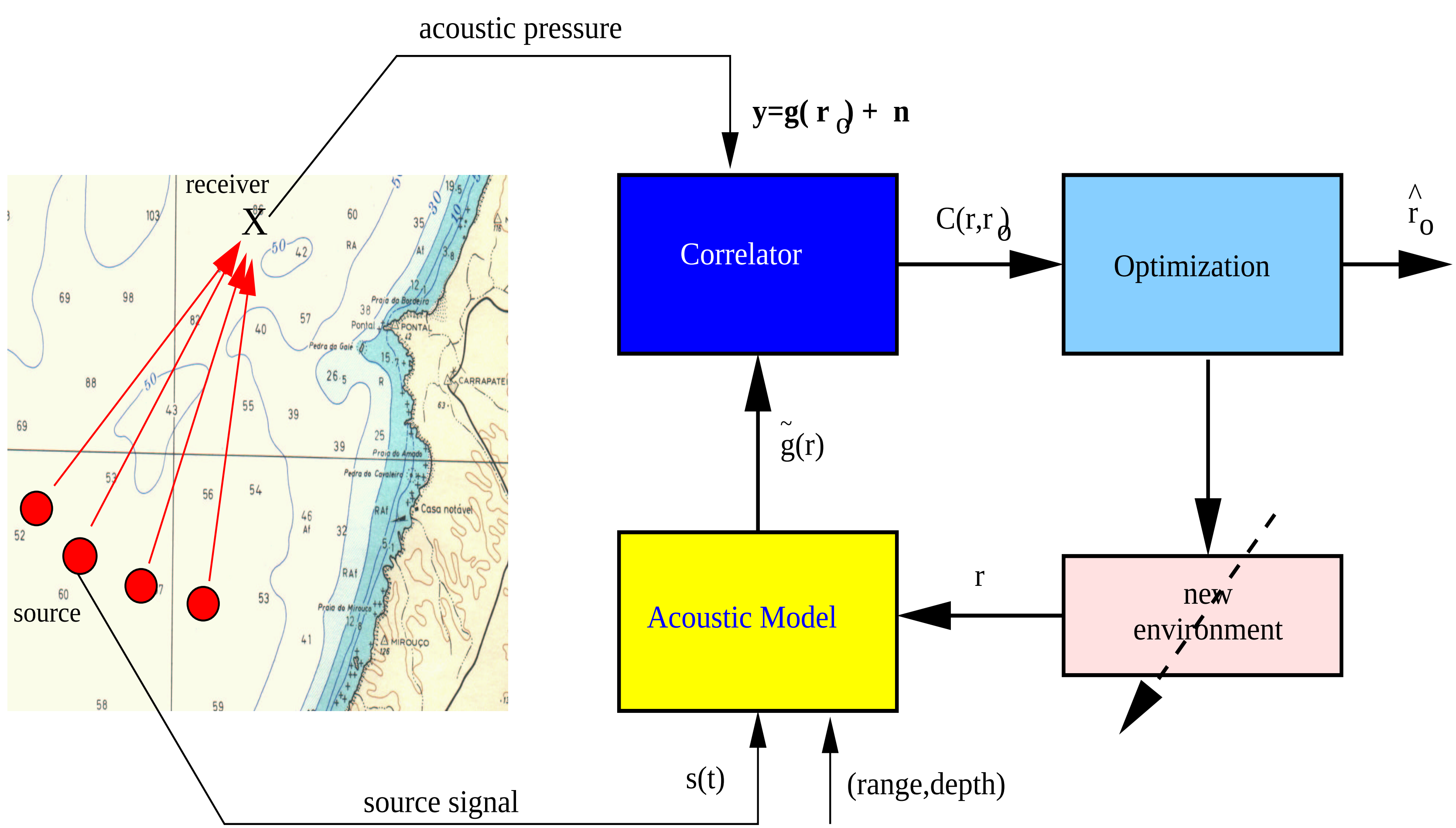
Outline

- Model-Based Processing (MBP)
- the problem
- the inversion methods
- real world examples
- conclusions

Ocean Acoustic Tomography - synoptic



The problem

- **Implicit Model Conditioning**

Data: $y(\mathbf{r}_o) = g(\mathbf{r}_o) + n$

Model: $\tilde{g}(\mathbf{r})$

$$\hat{\mathbf{r}}_o = \min_{\mathbf{r}} \quad \mathcal{F}[y(\mathbf{r}_o), \tilde{g}(\mathbf{r})]$$

- **Model with random components**

Data: $y(\mathbf{r}_o) = g(\mathbf{r}_o) + n$

Model: $m(\mathbf{r}) = \alpha\tilde{g}(\mathbf{r})$

α and $\mathbf{r}$ random

Forward models

$$\nabla^2 p + \frac{\omega^2}{c^2(z)} p = 0$$

- **Ray solution**

$$\tau = \int_\Gamma \frac{ds(z,r)}{c(z)}$$

$$p = p_o \sum_{i=1}^{I} \frac{1}{4\pi s_i} \exp\left(j\omega\tau_i\right)$$

<u>Forward models</u>(cont.)

- **Normal-mode solution**

$$p = p_o \sum_{m=1}^{M} \frac{\phi_m(z_o)\phi_m(z)}{\sqrt{k_m r}} \exp\left(ik_m r - \alpha_m r\right)$$

$$\frac{d^2\phi_m}{dz^2} + \left[\frac{\omega^2}{c^2(z)} - k_m^2\right]\phi_m = 0$$

Perturbation inversion example

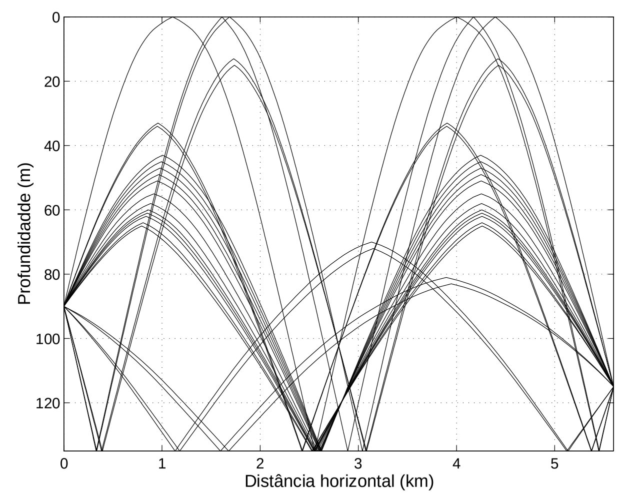
INTIMATE sea trial June 96 - Nazaré, Portugal

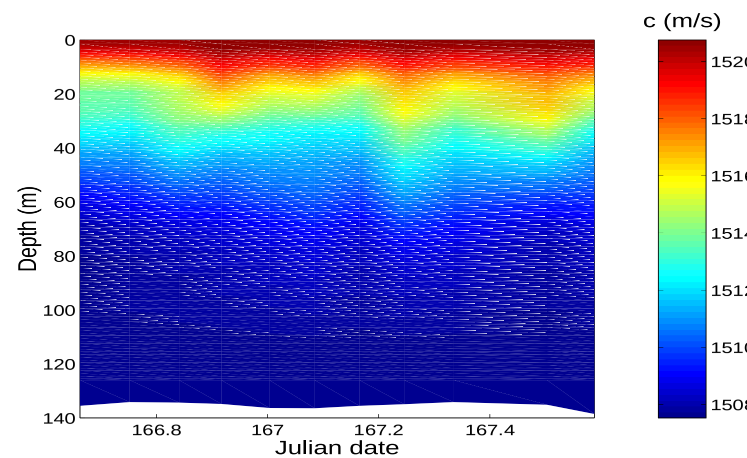
$$\Delta\tau \approx -\int_\Gamma \frac{\delta c(z)}{c_o^2(z)} ds$$

$$\boldsymbol{\Delta}\tau = \mathbf{E}\delta\mathbf{c} + \mathbf{n}$$

$$\delta\mathbf{c} = \boldsymbol{\Psi}\alpha$$

$$\alpha = \left[\boldsymbol{\Psi}^t\mathbf{E}^t\mathbf{E}\boldsymbol{\Psi}\right]^{-1}\boldsymbol{\Psi}^t\mathbf{E}^t\boldsymbol{\Delta}\tau$$





Forward model-based example

Data model:

$$y(t, \mathbf{r}_o) = x(t, \mathbf{r}_o) + n(t)$$
$$x(t, \mathbf{r}_o) = g(t, \mathbf{r}_o) * s(t)$$

Mean square error estimator:

$$\hat{\mathbf{r}}_o = \min_{\mathbf{r}} E[\|\mathbf{r} - \mathbf{r}_o\|^2]$$

The Matched-Filter (or correlator-receiver):

$$H(\omega, \mathbf{r}_o) = H_o \frac{G^*(\omega, \mathbf{r}_o) S^*(\omega)}{P_{nn}(\omega)} e^{-j\omega\tau}$$

Forward model-based example (cont.)

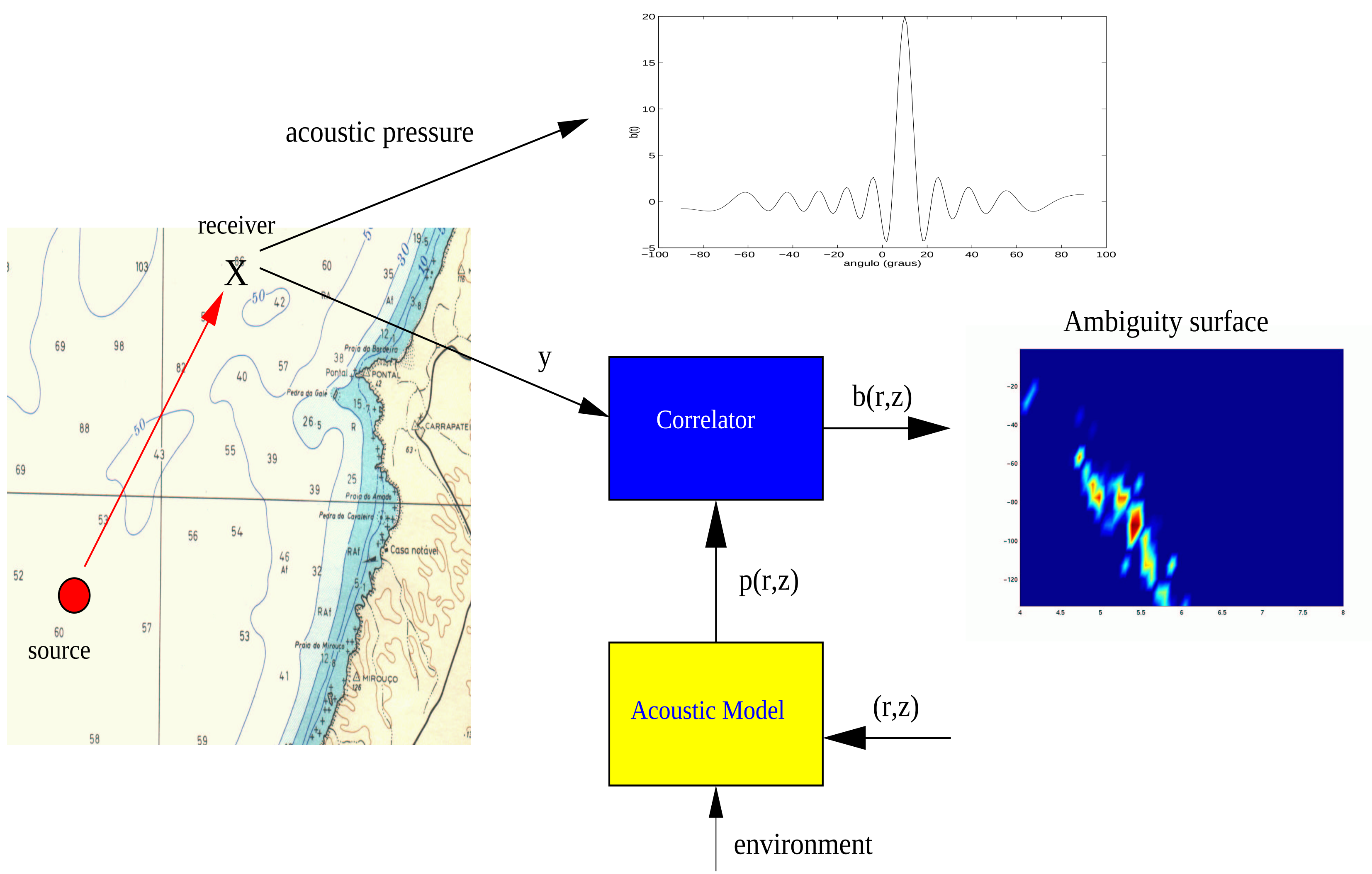
⇒ **Neyman-Pearson detector** (WGN)
⇒ **maximizes the Signal-to-Noise Ratio (SNR)**

$$\rho(t,\mathbf{r}) = \frac{1}{2\pi} \frac{\left| \int_\Omega H(\omega,\mathbf{r}) G(\omega,\mathbf{r}) S(\omega) e^{j\omega t} d\omega \right|^2}{\int_\Omega \left| H(\omega,\mathbf{r}) \right|^2 P_{nn}(\omega) d\omega}$$

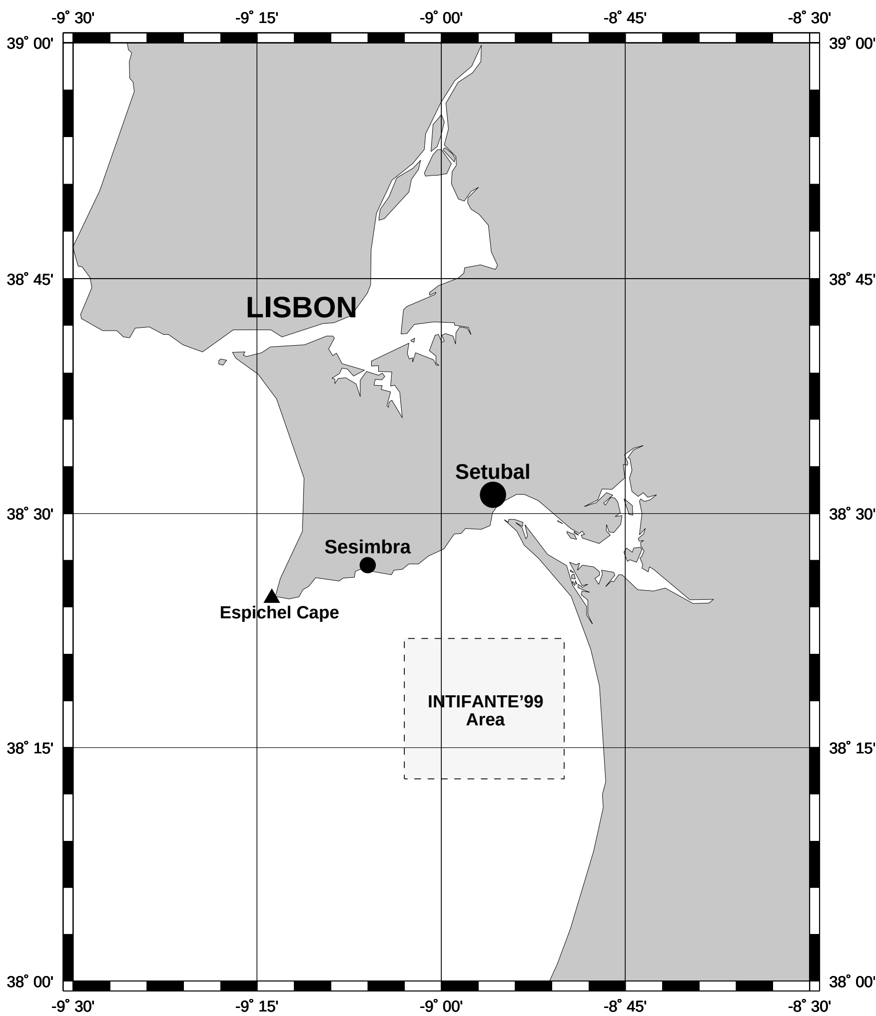
or the SNR estimate

$$\hat{\rho}(t,\mathbf{r}) = E\left[\frac{\left| \int_\Omega \tilde{G}^*(\omega,\mathbf{r}) S^*(\omega) Y(\omega,\mathbf{r}_o) e^{j\omega(t-\tau)} d\omega \right|^2}{\int_\Omega |\tilde{G}^*(\omega,\mathbf{r}) S^*(\omega)|^2 P_{nn}(\omega) d\omega} \right]$$

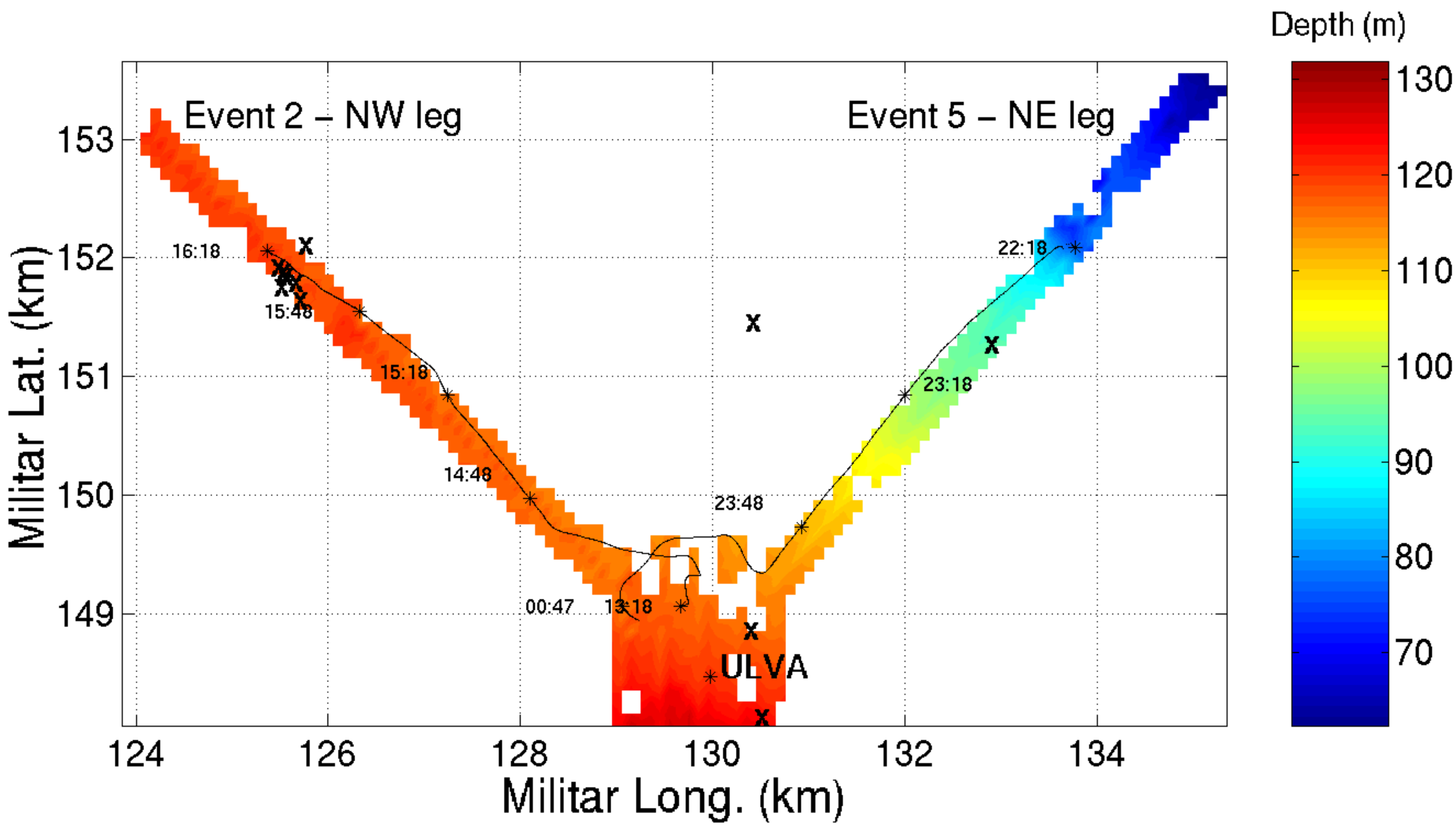
Forward model based - synoptic



INTIFANTE'00 Experimental Site



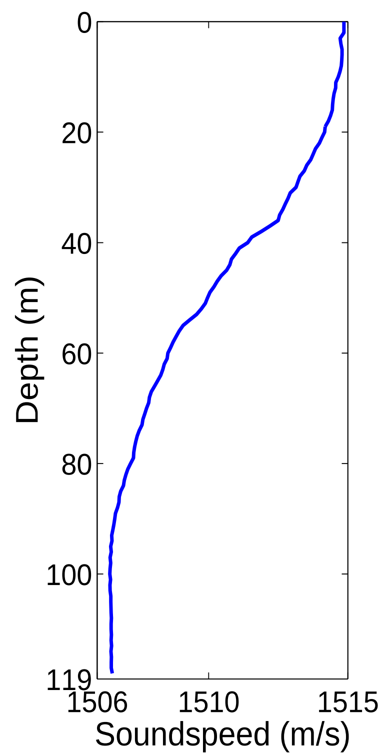
Events 2 and 5: bathymetry and runs

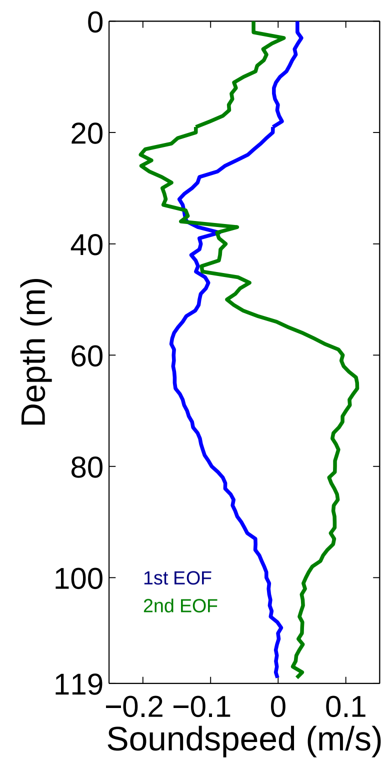


Empirical Orthogonal Functions

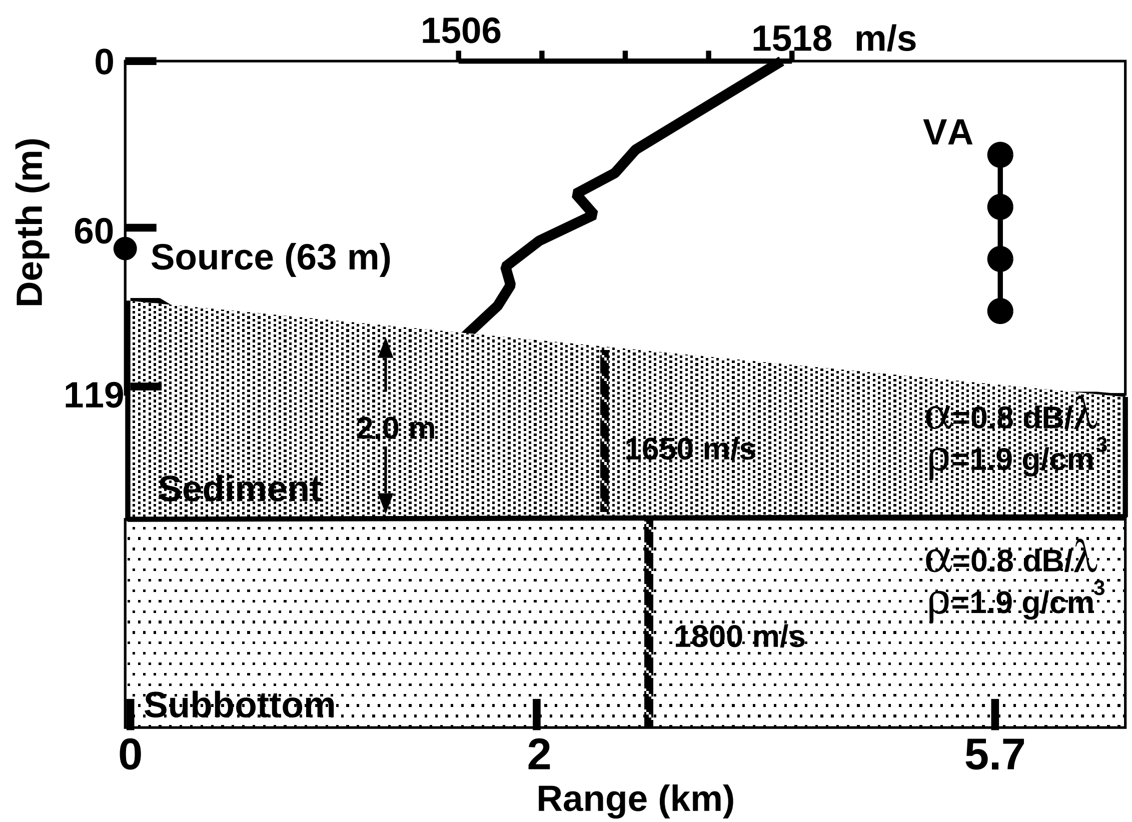
$$\mathbf{c}_{\mathrm{EOF}} = \hat{\mathbf{c}} + \sum_{n=1}^{N} \alpha_n \mathbf{u}_n$$

$$\hat{N} = \min_{N}\left\{\frac{\sum_{n=1}^{N} \lambda_n^2}{\sum_{m=1}^{M} \lambda_m^2} > 0.8\right\}$$

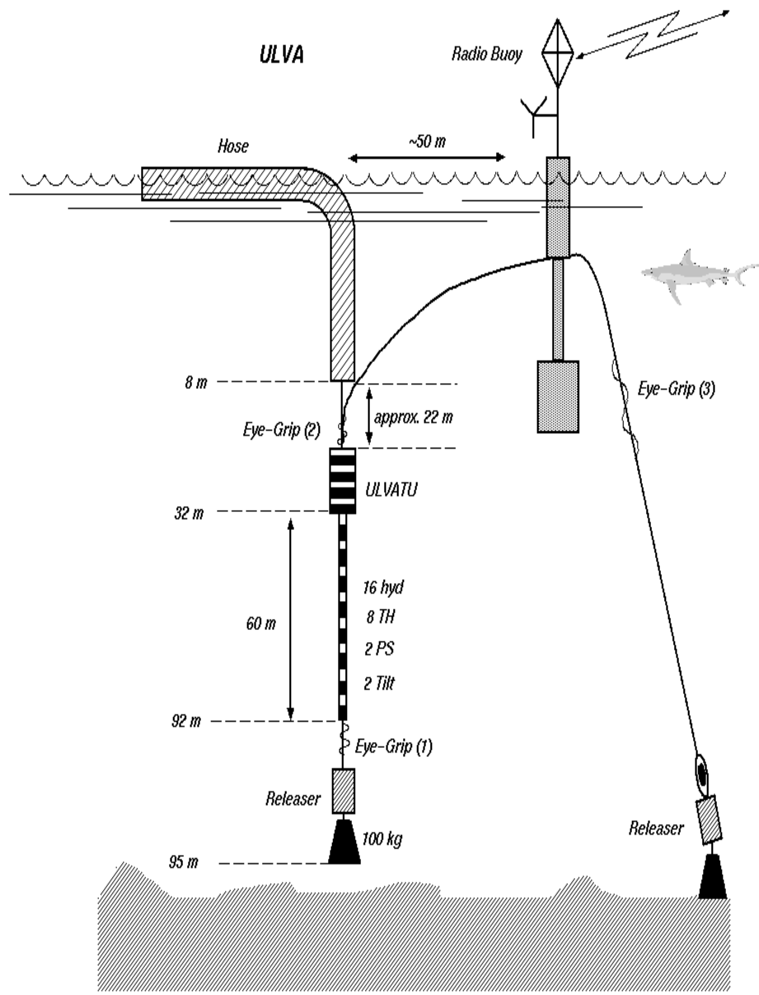




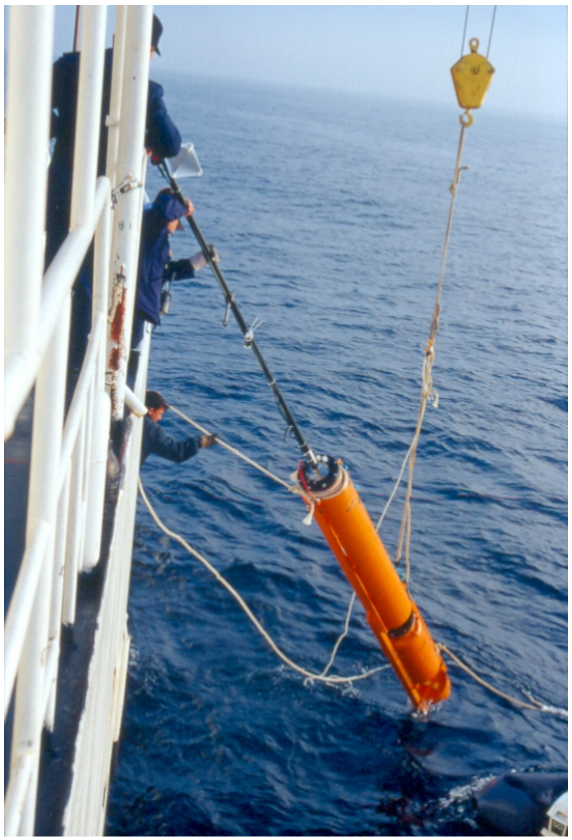
Physical model - NW/NE track

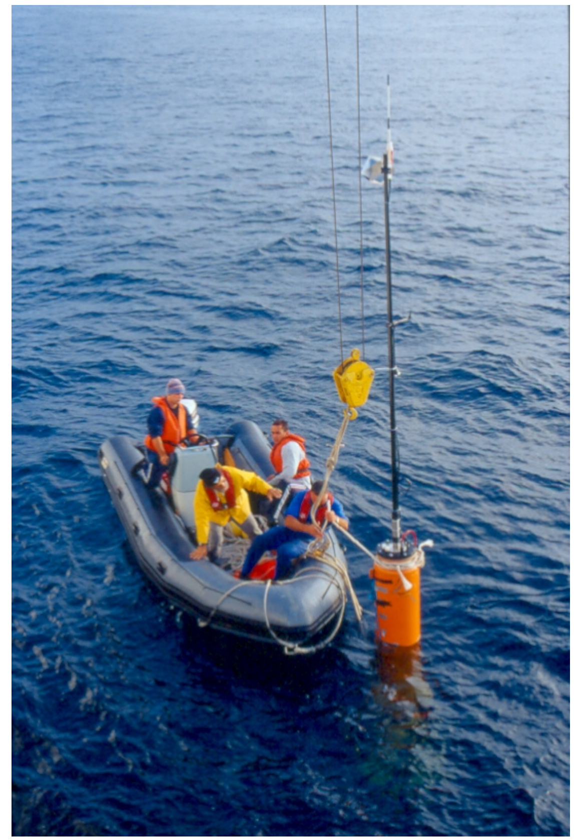


Vertical Line Array

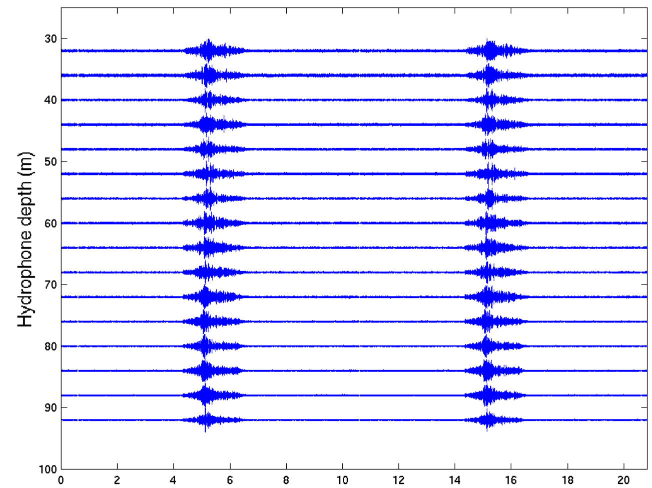


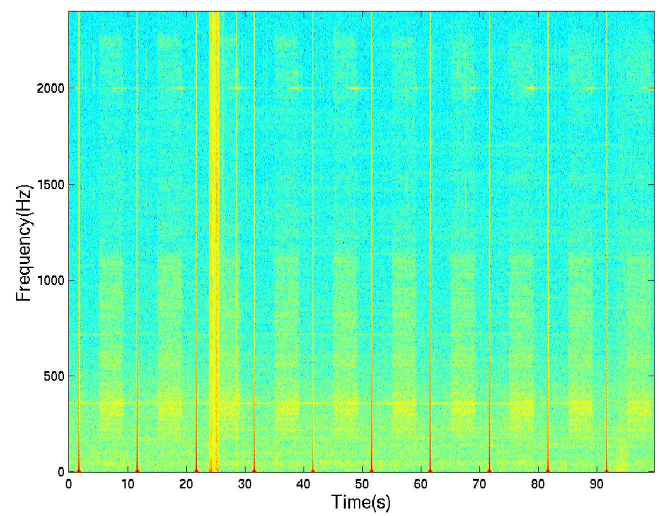
Radio Buoy: deployment and setup



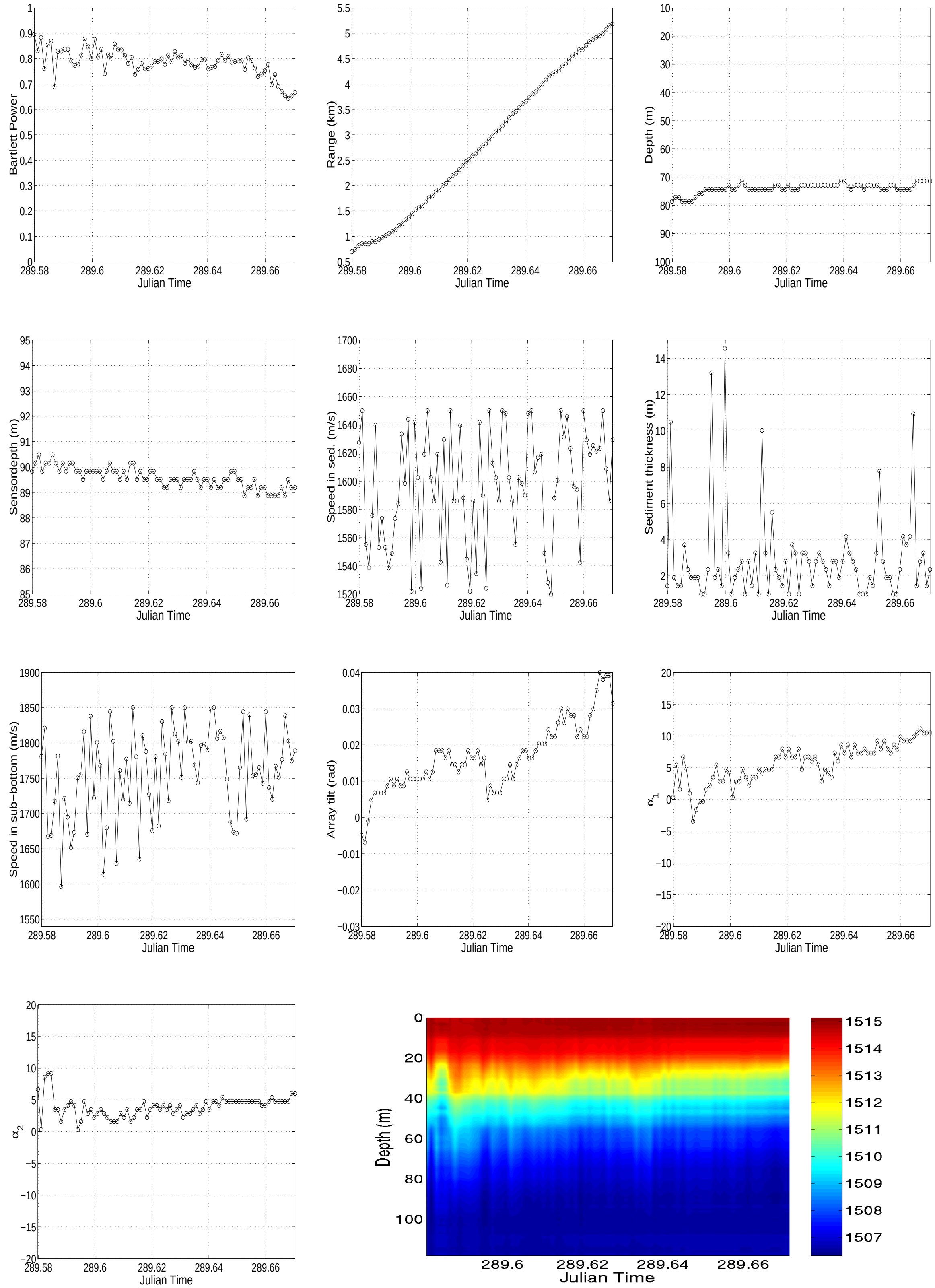


Sound source and emitted signals

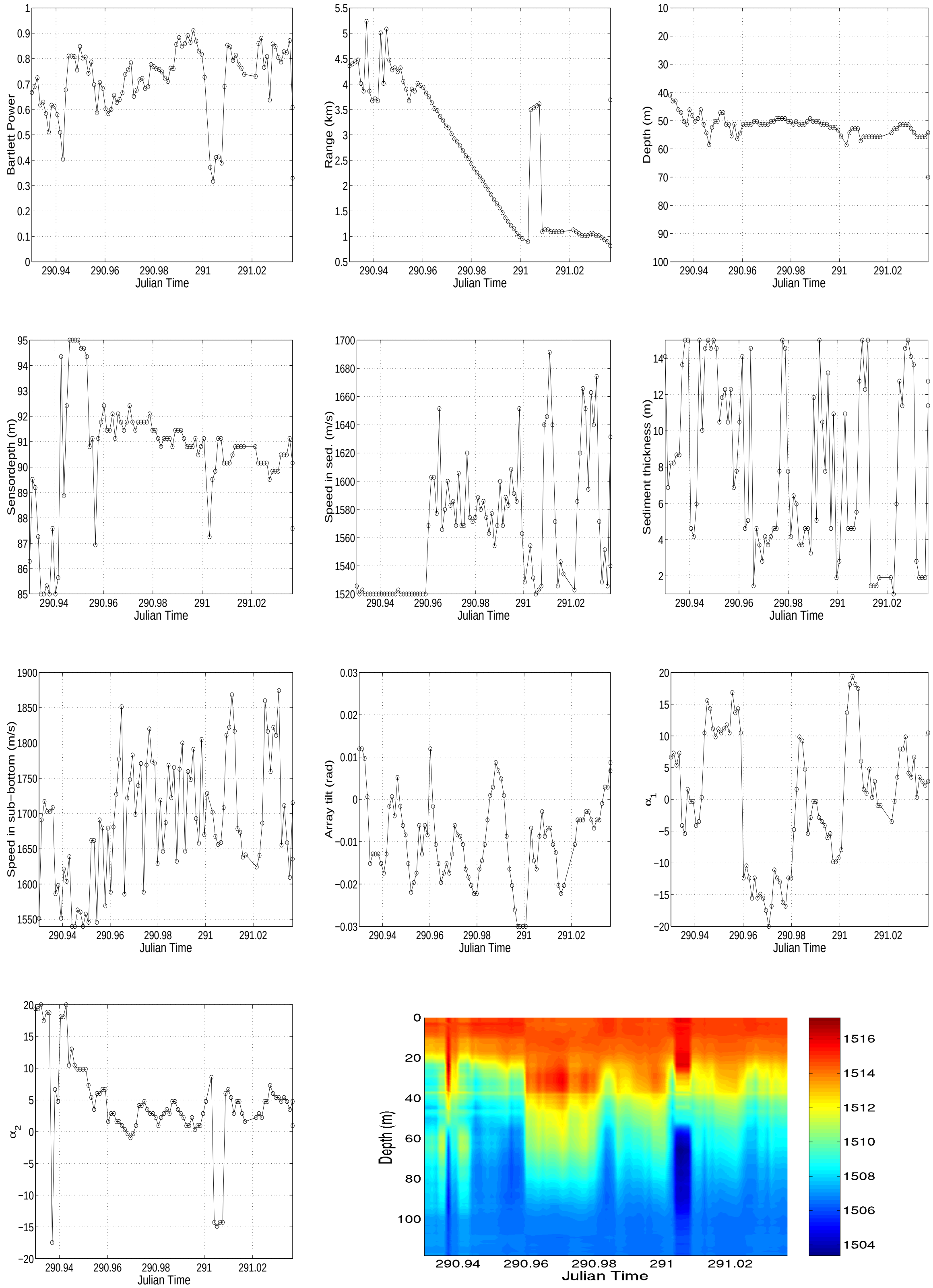




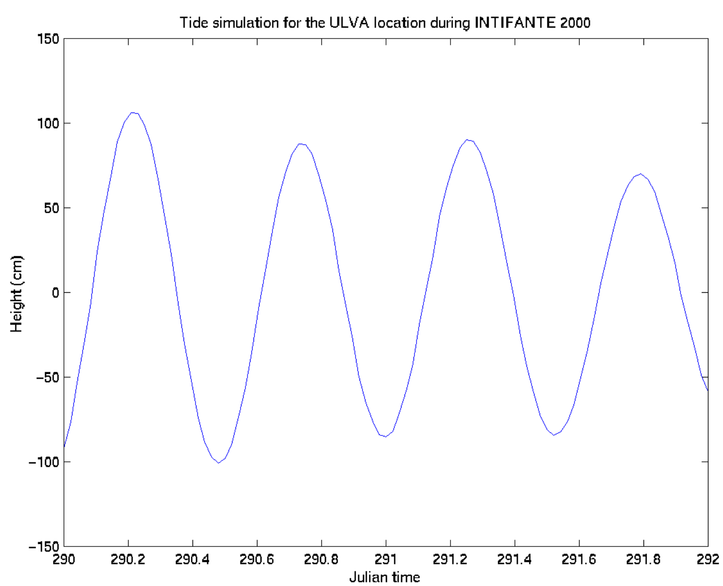
Inversion results for Event 2

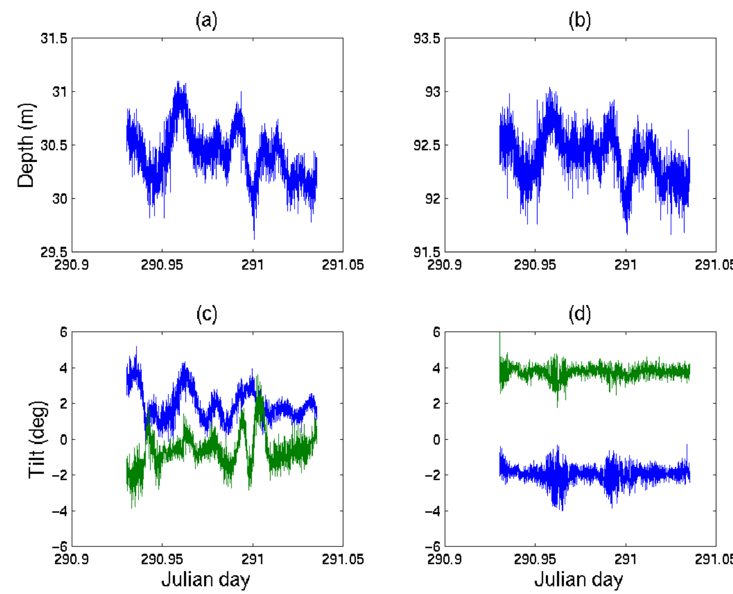


Inversion results for Event 5



Tide evolution and VA moving





Inverse modelling example

Training

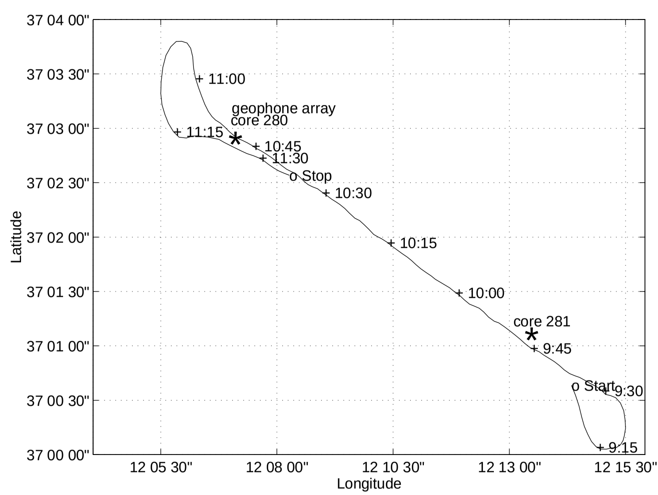
The pair $\{r_j, x(r_j)\}$ allows for calculating coefficients c_{ij} of the neural net (RBF) network parameterized by functions ϕ

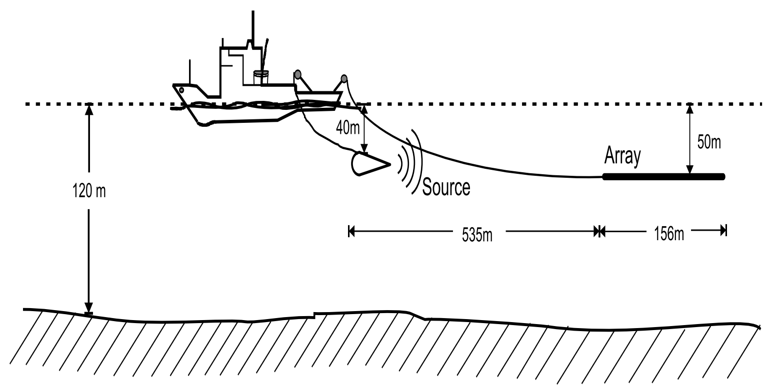
Data Inversion

Given $y(\mathbf{r}_o)$ calculate $\hat{\mathbf{r}}_o = \sum_{j=1}^{N} c_{ij}\phi(\|y(\mathbf{r}_o) - x(r_j)\|)$

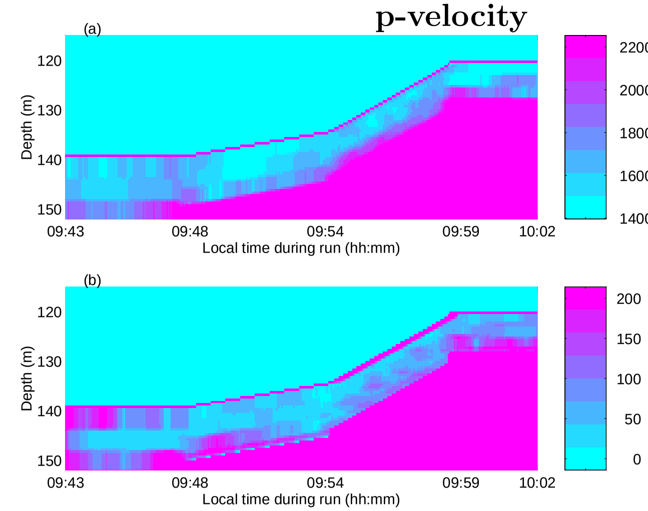
Inverse modelling example(cont.)

ADVENTURE BANK sea trial 1994 - Sicily





p-velocity



s-velocity